Math 131 Fall, 2008 Review - Hour Exam 2

1. **Review materials** covered in **Sections 2.1-2.7** and **lecture notes** for these sections, **homework assignments 7-11**, and **quizzes 4-6**.
2. **Definition of derivative**:
   a. Definition of derivative of $f(x)$ at $x = a$, and definition of derivative of $f(x)$.
   (i) State the definition: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ ($f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$) if the limit exists.
   (ii) Compute $f'(a)$ or $f'(x)$ by definition. Examples:
   (1) $f(x) = 2x^2 - 3x + 1,\ a = -1$ (2) $f(x) = \sqrt{3x+1},\ a = 1$ (3) $f(x) = \frac{3}{x+1},\ a = 2$
   b. Describe mathematically by definition why $f(x)$ is NOT differentiable at $x = a$:
   (i) f is not continuous at $x = a$; (ii) $\lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h} \neq \lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h}$; or
   (iii) $\lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h} = -\infty$ or $+\infty$ or $\lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h} = -\infty$ or $+\infty$
   Examples: Problem 4 in HW8 and Problem 2 on Quiz 5.
3. **Differentiation rules**:
   a. $\frac{d}{dx}[cf(x)] = cf'(x)$ b. $\frac{d}{dx}[cf(x) \pm Cg(x)] = cf'(x) \pm Cg'(x)$ c. $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
   d. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ e. $\frac{d}{dx}[(g(x))^r] = r(g(x))^{r-1}g'(x)$ f. $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

   Basic Differentiation rules formulas:

$f(x)$	x^r	e^x	a^x	$\ln(x)$	$\log_a(x)$	$\sin x$	$\cos x$	$\tan x$	$\cot x$	$\sec x$	$\csc x$
$f'(x)$	$r\,x^{r-1}$	e^x	$a^x \ln a$	$\frac{1}{x}$	$\frac{1}{x\ln a}$	$\cos x$	$-\sin x$	$\sec^2 x$	$-\csc^2 x$	$\tan x \sec x$	$-\cot x \csc x$

   Examples:
   (i) Derive the derivative of $\tan(x)$ or $\cot(x)$ by derivatives of $\sin(x)$ and $\cos(x)$ and the quotient rule.
   (ii) Find $f'(x)$:
   (1) $f(x) = 5x^{2008} - \frac{3}{\sqrt{x^3}} - \sqrt[3]{x^2} + \frac{2}{x^4} + \pi^x + x^\pi$ (2) $f(x) = \left(\frac{1}{x} + x\right)\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)$ (Power Rule only)
   (3) $f(x) = \frac{(x^3+4)^5}{8}$ (4) $f(x) = \frac{6x - 2/x}{x^2 + \sqrt{x}}$ (5) $f(x) = \left(x^2 + \sqrt{2}\right)\left(\frac{x^2-1}{x^2+x}\right)$ (6) $f(x) = \sqrt{4x^2 + (8-x^2)^2}$
   (7) $f(x) = \sin(3x) - 3\cos(\pi x)$ (8) $f(x) = \sqrt{\sin^2(x) + \sec(2x)}$ (9) $f(x) = 2\tan\left(\sqrt{x^2+1}\right)$
   (10) $f(x) = \cos^4(\pi x) - 3\sec(x^4)$ (11) $f(x) = 2e^{-4x} + 2^{1-x} - (0.12)^x$ (12) $f(x) = xe^{2x}$
   (13) $f(x) = x^3 \ln(x)$ (14) $f(x) = xe^{-\sin(x)}$ (15) $f(x) = 4x^2 \cot(\pi x)\sec(3x)$ (16) $f(x) = x\ln(\cos(x^2))$
   (iii) Find $f''(t)$ where $f(t) = \frac{60t}{\sqrt{t^2+1}}$. Find $g^{(77)}(x)$ where $g(x) = \cos(x)$.
4. **Applications of derivatives**:
   a. Derivative as the slope of a tangent line to the curve $y = f(x)$ at $x = a$:
   Slope $m = f'(a)$, the equation $y - f(a) = f'(a)(x - a)$. A tangent line is horizontal if $f'(a) = 0$.
   b. Velocity $v(t)$ and acceleration $a(t)$ of a position function $s(t)$: $v(t) = s'(t),\ a(t) = v'(t) = s''(t)$.
   The object is moving forward (backward) at $t = t_0$ if $v(t_0) > 0$ ($v(t_0) < 0$).
   Examples: (a) Find the equation of the tangent line to the curve $y = f(x)$ at $x = a$:
   (i) $f(x) = \frac{6}{x^2+4},\ a = -2$ (ii) $f(x) = \tan(3x),\ a = 0$ (iii) $f(x) = \frac{x^2}{g(x)},\ a = 1,\ g(1) = 2,\ g'(1) = -3$
   (b) (i) Find velocity $v(t)$ of $s(t) = \frac{\cos(t)}{t}$. Is object moving forward when $t = \pi$?
   (ii) Find velocity $v(t)$ and acceleration $a(t)$ of (1) $s(t) = \sqrt{x^2+1}$ (2) $s(t) = e^{-x}\sin(2x)$.