Math 131 Spring, 2008 Review - Hour Exam 3
Sections (2.3)-(2.9), (3.1), (3.3), (3.4), (3.8), HW 12-15, Quiz 7-9

1. **All differentiation formulas and rules introduced in Chapter 2.** (**Review for Hour Exam 2**).
Example: Find $f'(x)$ where (i) $f(x) = x^x$ (ii) $f(x) = (1.12)^{\tan(\sqrt{x})}$ (iii) $f(x) = \sin^{\pi}(x)$

2. **Implicit Differentiation**: compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for a given equation $F(x,y) = G(x,y)$.
Examples:
(**1**) Find $\frac{dy}{dx}$ where (i) $3xy^3 - 4x = 10y^2$ (ii) $\cos(xy) + x^2 = x^3y^2 - 3$ (iii) $\sqrt{x+y} - 4x^2 = y$ (iv) $e^{x^2y} - e^y = x$
(**2**) Find $\frac{d^2y}{dx^2}$ where (i) $\frac{dy}{dx} = \frac{y}{2xy+1}$ (ii) $\frac{dy}{dx} = \frac{3x^2y}{\sin(xy^2)}$
(**3**) Let $\frac{dy}{dx} = \frac{y-1}{2xy+1}$. (i) Find the equation of the tangent line to the curve y at $(-1,-3)$. (ii) If the curve has a horizontal tangent line at a point (a,b), then what is the value of b?

3. **Section 3.8**: Related rates. Examples: From Homework 13: Page 324: 6, 8, 10, 14.

4. **Derivatives of Inverse Trigonometric Functions**.

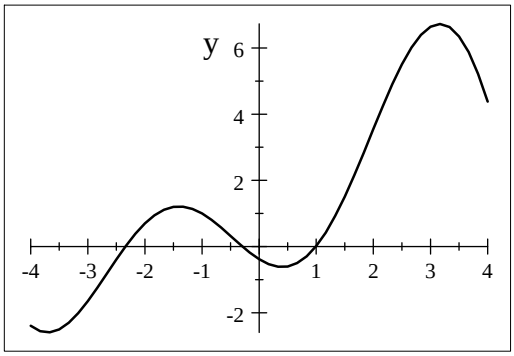
$f(x)$	$f'(x)$
$\sin^{-1}(g(x))$	$\frac{g'(x)}{\sqrt{1-(g(x))^2}}$
$\tan^{-1}(g(x))$	$\frac{g'(x)}{1+[g(x)]^2}$
$\sec^{-1}(g(x))$	$\frac{1}{\lvert g(x)\rvert \sqrt{[g(x)]^2-1}}$

Example: Find $f'(x)$ where
(i) $f(x) = \sin^{-1}(x^3+1) + 4\sec^{-1}(x^4) - \sqrt{2+\tan^{-1}(x)}$.
(ii) $f(x) = \frac{x^2}{\cot^{-1}(x)} + \cos^{-1}(\sqrt{x})$

5. **Rolle's Theorem, and the Mean-Value Theorem**.
Find **graphically** and **algebraically** the value of c satisfying the conclusion of Rolle's Theorem or the Mean-Value Theorem.

Graphically find value(s) of c :

Example:



$y = f(x)$

Algebraically find value(s) of c :
(i) $f(x) = x^2 + 1$, (1) $[0,2]$ (2) $[-1,1]$
(ii) $f(x) = x^3 + x^2$, (1) $\left[-1,\ 1\right]$ (2) $\left[-1,\ 0\right]$

6. **Section 3.1**:

Linear approximation of $f(x)$ at $x = a$: $L(x) = f(a) + f'(a)(x-a)$, $f(x) \approx L(x) = f(a) + f'(a)(x-a)$

Example: (i) $f(x) = \sqrt[3]{x+1}$, $a = 0$, $\sqrt[3]{1.01} \approx L(1.01)$ (ii) $f(x) = \sin(x)$, $a = 0$, $\sin(0.01) \approx L(0.01)$

7. **Section 3.3-3.4**:

(1) **Properties** of $f(x)$ from $f'(x)$ and the First Derivative Test:

$f(x)$	$f'(x)$
Critical number at c	(1) c is in domain of f (2) (i) $f'(c) = 0$; or (ii) $f'(c)$ is not defined
Increasing on (a,b)	$f'(x) > 0$ on (a,b)
Decreasing on (a,b)	$f'(x) < 0$ on (a,b)
$(c,f(c))$ a **local maximum**	$f'(x)$ changes from + to –
$(c,f(c))$ a **local minimum**	$f'(x)$ changes from – to +

Examples:

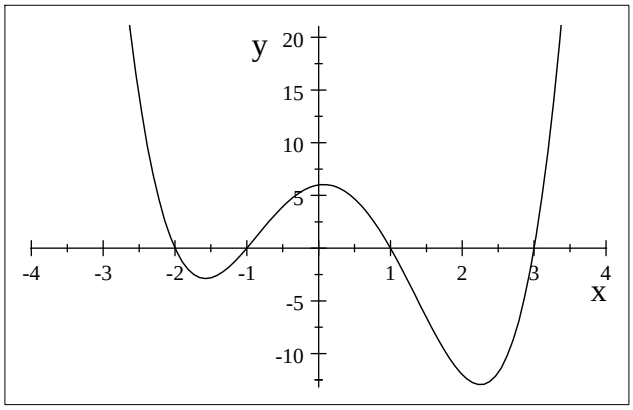
For the following functions, find

(a) all critical numbers;

(b) Determine where f is increasing and where f is decreasing;

(c) local maximum point and local minimum point.

(i) $f(x) = x^2e^{-3x}$
(ii) $f(x) = x^3 - 3x^2 + 3x$
(iii) $f(x) = x^5 - 5x^2 + 1$
(iv) $f(x) = (x-1)^{1/3}$
(v) the graph of $f'(x)$ is given at the left:
(vi) $f'(x) = x^2(x-3)(x^2-2)$
(vii) $f'(x) = x\,(x-3)e^{-x}$



(2) **Absolute maximum and minimum of** $f(x)$ over $[a,b]$:

Steps:

(a) Find all critical numbers c of $f(x)$ in (a,b);

(b) Compare values all $f(c)$, $f(a)$ and $f(b)$ to determine the absolute maximum values and minimum value of $f(x)$.

Examples:

(i) Find absolute maximum and minimum of $f(x) = x^4 - 4x^2 + 3$ over (A) $[-3,1]$ (B) $[-1,3]$

(ii) Find absolute maximum and minimum of $f(x) = x^2e^{-4x}$ over (A) $[-2,0]$ (B) $[0,4]$