Math 131 Spring, 2008  Review - Hour Exam 3
Sections (2.3)-(2.9), (3.1), (3.3), (3.4), (3.8), HW 12-15, Quiz 7-9

1. All differentiation formulas and rules introduced in Chapter 2. (Review for Hour Exam 2).

Example: Find $f'(x)$ where (i) $f(x) = x^4$  (ii) $f(x) = (1.12)^{\tan(x^2)}$  (iii) $f(x) = \sin^2(x)$

2. Implicit Differentiation: compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for a given equation $F(x,y) = G(x,y)$.

Examples:
(i) Find $\frac{dy}{dx}$ where (i) $3xy^3 - 4x = 10y^2$  (ii) $\cos(xy) + x^2 = x^3y^2 - 3$  (iii) $\sqrt{x+y} - 4x^2 = y$  (iv) $e^{x^2y} - e^{-y} = x$
(ii) Find $\frac{d^2y}{dx^2}$ where (i) $\frac{dy}{dx} = \frac{y}{2xy+1}$  (ii) $\frac{dy}{dx} = \frac{3x^2y}{\sin(xy^2)}$
(iii) Let $\frac{dy}{dx} = \frac{y-1}{2xy+1}$. (i) Find the equation of the tangent line to the curve $y$ at $(-1,-3)$. (ii) If the curve has a horizontal tangent line at a point $(a,b)$, then what is the value of $b$?


4. Derivatives of Inverse Trigonometric Functions.

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$f'(x)$</th>
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</thead>
<tbody>
<tr>
<td>$\sin^{-1}(g(x))$</td>
<td>$\frac{g'(x)}{\sqrt{1-(g(x))^2}}$</td>
</tr>
<tr>
<td>$\tan^{-1}(g(x))$</td>
<td>$\frac{g'(x)}{1+[g(x)]^2}$</td>
</tr>
<tr>
<td>$\sec^{-1}(g(x))$</td>
<td>$\frac{1}{</td>
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</table>

Examples: Find $f'(x)$ where
(i) $f(x) = \sin^{-1}(x^3 + 1) + 4\sec^{-1}(x^4) - x + \tan^{-1}(x)$
(ii) $f(x) = \frac{x^2}{\cot^{-1}(x)} + \cos^{-1}(\sqrt{x})$

5. Rolle’s Theorem, and the Mean-Value Theorem.

Find graphically and algebraically the value of $c$ satisfying the conclusion of Rolle’s Theorem or the Mean-Value Theorem.

Graphically find value(s) of $c$:

Example:

Algebraically find value(s) of $c$:
(i) $f(x) = x^2 + 1$, (1) $[0,2]$ (2) $[-1,1]$
(ii) $f(x) = x^3 + x^2$, (1) $[-1, 1]$ (2) $[-1, 0]$
6. Section 3.1:
Linear approximation of \( f(x) \) at \( x = a \):
\[
L(x) = f(a) + f'(a)(x-a), \quad f(x) \approx L(x) = f(a) + f'(a)(x-a)
\]
Example: (i) \( f(x) = \sqrt{x+1}, \; a = 0, \; \sqrt{1.01} \approx L(1.01) \) (ii) \( f(x) = \sin(x), \; a = 0, \; \sin(0.01) \approx L(0.01) \)

7. Section 3.3-3.4:
(1) Properties of \( f(x) \) from \( f'(x) \) and the First Derivative Test:

<table>
<thead>
<tr>
<th>Critical number at ( c )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ( c ) is in domain of ( f )</td>
<td>(2) (i) ( f'(c) = 0 ); or</td>
</tr>
<tr>
<td></td>
<td>(ii) ( f'(c) ) is not defined</td>
</tr>
<tr>
<td>Increasing on ( (a,b) )</td>
<td>( f'(x) &gt; 0 ) on ( (a,b) )</td>
</tr>
<tr>
<td>Decreasing on ( (a,b) )</td>
<td>( f'(x) &lt; 0 ) on ( (a,b) )</td>
</tr>
<tr>
<td>( (c,f(c)) ) a local maximum</td>
<td>( f'(x) ) changes from + to -</td>
</tr>
<tr>
<td>( (c,f(c)) ) a local minimum</td>
<td>( f'(x) ) changes from - to +</td>
</tr>
</tbody>
</table>

Examples:
For the following functions, find
(a) all critical numbers;
(b) Determine where \( f \) is increasing and where \( f \) is decreasing;
(c) local maximum point and local minimum point.

(i) \( f(x) = x^2e^{-3x} \)
(ii) \( f(x) = x^3 - 3x^2 + 3x \)
(iii) \( f(x) = x^5 - 5x^2 + 1 \)
(iv) \( f(x) = (x - 1)^{1/3} \)
(v) the graph of \( f'(x) \) is given at the left:
(vi) \( f'(x) = x^2(x - 3)(x^2 - 2) \)
(vii) \( f'(x) = x(x - 3)e^{-x} \)

(2) Absolute maximum and minimum of \( f(x) \) over \([a,b] \):
Steps:
(a) Find all critical numbers \( c \) of \( f(x) \) in \((a,b) \);
(b) Compare values all \( f(c), f(a) \) and \( f(b) \) to determine the absolute maximum values and minimum value of \( f(x) \).

Examples:
(i) Find absolute maximum and minimum of \( f(x) = x^4 - 4x^2 + 3 \) over \((A) \; [-3, 1] \) \( (B) \; [-1, 3] \)
(ii) Find absolute maximum and minimum of \( f(x) = x^2e^{-4x} \) over \((A) \; [-2, 0] \) \( (B) \; [0, 4] \)