1. Reading materials:
   Textbook - Page 286-292
   (1) Definitions 5.1 and Theorem 5.1, and Definition 5.2, and Theorem 5.2.
   (2) Examples 5.1, 5.2, 5.3, 5.4 and 5.5.
   (3) Lecture Notes on Section 3.5.

2. Graphs of $f(x)$, $g'(x)$ and $h''(x)$ for $-5 \leq x \leq 5$ are given below.
   
   a. Estimate graphically all possible inflection points $c$ of $f(x)$ in the interval $(-5, 5)$. Determine interval(s) on which
      (1) $f(x)$ is concave up and $f(x)$ is concave down.  (2) $f(x)$ is increasing and concave down.
   
   b. Estimate graphically all possible inflection points $c$ of $g(x)$ in the interval $(-5, 5)$. Determine interval(s) on which
      (1) $g(x)$ is concave up and $g(x)$ is concave down.  (2) $g(x)$ is increasing and concave down.
   
   c. Estimate graphically all possible inflection points $c$ of $h(x)$ in the interval $(-5, 5)$. Determine interval(s) on which
      (1) $h(x)$ is concave up and $h(x)$ is concave down.
      (2) If we also know $x = -4$, $x = -1$ and $x = 3$ are critical numbers of $h(x)$. Use the 2nd Derivative Test to classify each critical number as the location of a local maximum, local minimum or no conclusion.

b. Let the graph of $g'(x)$ in $(-\infty, \infty)$ be given below. Assume $g(x)$ is defined everywhere.
(1) Find all critical numbers of $g(x)$.
$g'(x) = 0, x = -5, -1, 2.6, 4.7$

(2) Determine all intervals on which $g(x)$ is decreasing.
$g'(x) < 0 : (-\infty, -5), (2.6, 4.7)$

(3) Find all local minimum points.
$g'(x)$ changes from - to +: $x = -5, 4.7$

(4) Find all inflection points of $g$.
Where the graph changes from inc. to dec. or dec. to inc.
$x = -3.5, x = -1, x = 1.25, x = 3.75$

(5) Find the intervals on which $g$ is concave down.
Intervals where the graph is decreasing.
$(-3.5, -1), (1.25, 3.75)$

2. Let the graph of $h''(x)$ in $(-\infty, \infty)$ be given below.

(1) Find all inflection points of $h$.
Where the graph changes from + to - or from - to +:
$x = -5, x = 2.6, x = 4.8$

(2) Find the intervals on which $h$ is concave down.
Where the graph is negative:
$(-\infty, -5), (2.6, 4.8)$

(3) If we also know that $h'(-3) = 0$,
$h'(-1) = 0$ and $h'(4) = 0$, determine if $x = -3$, $x = -1$, and $x = 4$ are local minimum point, local maximum point or neither.

Apply the 2nd Derivative Test,
$h''(-3) > 0, h(-3)$ is a local minimum point.
$h''(-1) = 0$, no conclusion
$h''(4) < 0, h(4)$ is a local maximum point.

3. **Page 293**: Turn in the ones with *.

   **a.** For each of the following problem, use Theorem 5.1 to determine algebraically intervals where $f(x)$ is concave up and concave down.

   1-8: *2, *4, *8
For Problem 1-8, we first determine where \( f''(x) \) is positive and where \( f''(x) \) is negative.

2. \( f(x) = x^4 - 6x^2 + 2x + 3, \) \( f'(x) = 4x^3 - 12x + 2, \) \( f''(x) = 12x^2 - 12 \)

Set \( f''(x) = 0: \) \( 12(x^2 - 1) = 12(x-1)(x+1) = 0, x = \pm 1 \)

Check signs of \( f''(x) \) over \((-\infty, -1), (-1, 1), (1, \infty)\):

\[
\begin{array}{c|c|c|c}
& (-\infty, -1) & (-1, 1) & (1, \infty) \\
\hline
\text{sign of } f''(x) & + & - & + \\
\end{array}
\]

Hence, \( f(x) \) is concave up on \((-\infty, -1) \) and \((1, \infty) \) and concave down on \((-1, 1) \).

4. \( f(x) = x + 3(1 - x)^{1/3}, \) \( f'(x) = 1 - (1 - x)^{-2/3}, \) \( f''(x) = -\frac{2}{3} (1 - x)^{-5/3} = -\frac{2}{3 \sqrt[3]{(1 - x)^5}} \)

\( D_f = (-\infty, \infty), D_{f''} = (-\infty, 1) \cup (1, \infty) \)

\( f''(x) \neq 0 \) and \( f''(x) \) is not defined at \( x = 1 \). Check sign changes of \( f''(x) \) on \((-\infty, 1) \) and \((1, \infty)\):

\[
\begin{array}{c|c|c}
& (-\infty, 1) & (1, \infty) \\
\hline
\text{sign of } f''(x) & - & + \\
\end{array}
\]

\( f(x) \) is concave up on \((1, \infty) \) and is concave down on \((-\infty, 1) \).

8. \( f(x) = xe^{-4x}, \) \( f'(x) = e^{-4x} - 4xe^{-4x} = e^{-4x}(1 - 4x) \)

\( f''(x) = -4e^{-4x}(1 - 4x) - 4e^{-4x} = -4e^{-4x}(1 - 4x + 1) = -8e^{-4x}(2 - 4x) = -8e^{-4x}(1 - 2x) \)

\( f''(x) = 0: \) \( 1 - 2x = 0, x = \frac{1}{2} \).

Check sign changes of \( f''(x) \) on \((-\infty, \frac{1}{2}), \ (\frac{1}{2}, \infty) \):

\[
\begin{array}{c|c|c}
& (-\infty, \frac{1}{2}) & (\frac{1}{2}, \infty) \\
\hline
\text{sign of } f''(x) & - & + \\
\end{array}
\]

\( f(x) \) is concave up on \(\left( \frac{1}{2}, \infty \right) \), and concave down on \((-\infty, \frac{1}{2}) \).

b. For each of the following problem, use Theorem 5.2 (the 2nd Derivative Test) to classify each critical number as the location of a local maximum, local minimum or no conclusion.

9-14: *10, 11, *12

10. \( f(x) = x^4 + 4x^2 + 1, \) \( f'(x) = 4x^3 + 8x = 4(x^2 + 2), f''(x) = 12x^2 + 8. \)

(1) Critical numbers: \( f'(x) = 0, x = 0. \)

(2) Apply the 2nd Derivative Test: \( f''(0) = 8 > 0, f(0) = 1 \) is a local minimum value.

12. \( f(x) = e^{-x^2}, \) \( f'(x) = -2xe^{-x^2}, \) \( f''(x) = -2e^{-x^2} - 2x^2 e^{-x^2} = -2e^{-x^2}(1 - 2x^2) \).

\( D_f = (-\infty, \infty), D_{f'} = (-\infty, \infty), D_{f''} = (-\infty, \infty) \)

(1) Critical numbers: \( f'(x) = 0, x = 0 \)

(2) Apply the 2nd Derivative Test: \( f''(0) = -2e^0(1 - 0) < 0, f(0) = 1 \) is a local maximum value.

c. 41-44: 42*, extra points 44.
42. \( f(0) = 2 \)
\( f'(x) \geq 0 \) for all \( x \), \( f'(0) = 1 \)
\( f''(x) > 0 \) for \( x < 0 \)
\( f''(x) < 0 \) for \( x > 0 \)

4. **Page 284**: Turn in the ones with *.
35-38: 38*, extra points 36.