Math 131 Spring, 2008 Homework 16-2 Due April 1 Name: ________________

Mathematics Quote: *As far as the laws of MATHEMATICS refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.* **Albert Einstein**

1. Reading materials:

Textbook - Page 308-316

(1) Review 3 steps to find the absolute maximum and absolute minimum of $f(x)$ on $[a,b]$ given in Section 3.3.

(2) Examples 7.1, 7.2, 7.3, 7.4 and 7.5.

(3) Lecture Notes on Section 3.7.

3. **Page 317**: Turn in the ones with *.

3, 4*, 9, 10*, 11, 12*, 15, 16*.

Extra points: 28, 32

4. Let the dimensions of the rectangular region be W and L. Then we know $2W+L = 96$ and $A = area = WL$.

Because $L = 96-2W$, $A(W) = W(96-2W) = 2W(48-W)$.

We want to maximize $A(W)$ for W in $[0,48]$.

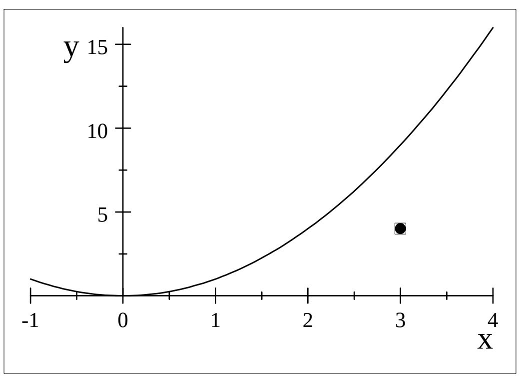
(1) Compute $A'(W)$: $A'(W) = 2[48-W+W(-1)] = 2(48-2W) = 4(24-W)$.

(2) Find the critical number of A : $A'(W) = 0,\ 24-W = 0,\ W = 24$.

(3) Check: $A(24) = 2(24)(48-24) = 1152,\ A(0) = 0,\ A(48) = 0$.

A is maximized when $W = 24$ and $L = 96-2(24) = 48$ ft and the maximum area is 1152 ft^2.

10.



$y = x^2,\ (3,4)$

Let (x,x^2) be on the curve $y = x^2$.

$f(x) = D^2(x,y) = (x-3)^2 + (x^2-4)^2$

We want to minimize $f(x)$ for x in $[0,\ 3]$

(1) $f'(x) = 2(x-3) + 2(x^2-4)(2x)$

$= 2(x-3+2x^3-8x) = 2(2x^3-7x-3)$

(2) Find $f'(x) = 0,\ 2x^3-7x-3 = 0,\ x = 2.05654529$.

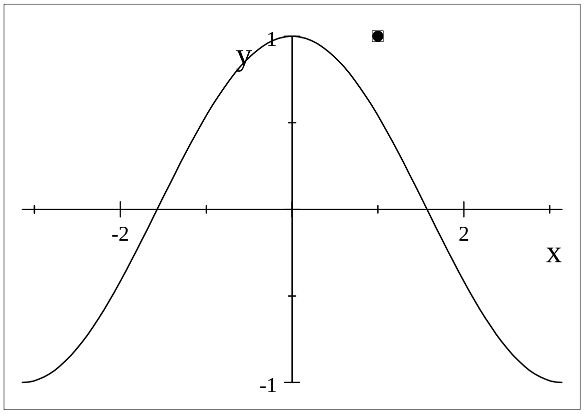
(3) Check:

$f'(0) = (0-3)^2 + (0-4)^2 = 25,\ f(2.056545) = (2.056545-3)^2 + (2.056545^2-4)^2 = 0.9427$

$f(3) = (3-3)^2 + (3^2-4)^2 = 25$

From $(3,4)$ to the point $x = 2.05654529$, and $y = (2.05654529)^2 = 4.22937853$, the distance is minimized.

12.



$y = \cos(x),\ (1,1)$

Let $(x, \cos(x))$ be on the curve $y = \cos(x)$.

$f(x) = D^2(x,y) = (x-1)^2 + (\cos(x)-1)^2$

We want to minimize $f(x)$ for x in $[0,\ 2]$

(1) $f'(x) = 2(x-1) + 2(\cos(x)-1)(-\sin(x))$

$= 2(x - 1 - \cos(x)\sin(x) + \sin(x))$

(2) Critical number: $f'(x) = 0$

$x = 0.789781396$

(3) Check $f(0.789781396) = (0.789781396-1)^2 + (\cos(0.789781396)-1)^2 = 0.131807515$

$f(0) = (0-1)^2 + (\cos(0)-1)^2 = 1,\ \ f(2) = (2-1)^2 + (\cos(2)-1)^2 = 3.00547186$

From (1.1) to the point $x = 0.530471426$ and $y = \cos(0.530471426) = 0.862568653$, the distance is minimized.

16. Let V be the volume of the box. Then $V(x) = x(12-x)(16-x) = 4x(6-x)(8-x)$.

We want to maximize $V(x)$ for x in $[0,6]$.

(1) $V'(x) = 4[(6-x)(8-x) + x(-1)(8-x) + x(6-x)(-1)] = 4(3x^2 - 28x + 48)$

(2) Critical numbers: $V'(x) = 0,\ \ 3x^2 - 28x + 48 = 0, x = 7.07036752, x = 2.26296582$

Solution is $x = 2.26296582$

(3) Check: $V(2.26296582) = 4(2.26296582)(6-2.26296582)(8-2.26296582) = 194.067358$

$V(0) = 0$ and $V(6) = 0$

Hence, when $x = 2.26296582$ the volume of the box is maximized.

3. Review:

Derivatives of the following functions:

$f(x) = x^r$

$f(x) = e^x,\ f(x) = a^x,\ \ f(x) = \ln(x),$

$f(x) = \sin(x),\ f(x) = \cos(x),\ f(x) = \tan(x),\ f(x) = \cot(x),\ \ f(x) = \sec(x),\ f(x) = \csc(x)$

$f(x) = \sin^{-1}(x),\ \ f(x) = \tan^{-1}(x),\ \ f(x) = \sec^{-1}(x)$

$f(x) = \ln(g(x))$

Question: If we know $f'(x) = x^{2008}$, what can say about $f(x)$?

Question: If we know the velocity function of a moving object at the time t is $v(t) = \sin(\pi t)$, what can we say about the position function of this moving object ?