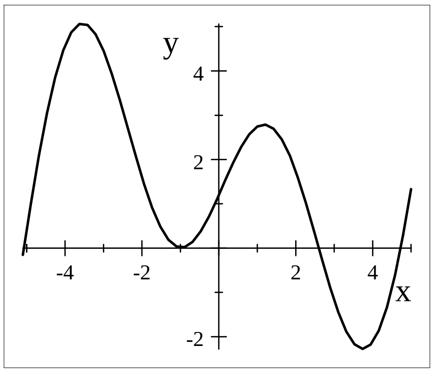
Math 131 Spring, 2008 Homework 16 Due March 20 Name: ________________

Mathematics Quote: *It can be of no practical use to know that Pi is irrational, but if we can know, it surely would be intolerable not to know.* - by Titchmarsh, E. C.

1. Reading materials:
Textbook - Page 286-292
(1) Definitions 5.1 and Theorem 5.1, and Definition 5.2, and Theorem 5.2.
(2) Examples 5.1, 5.2, 5.3, 5.4 and 5.5.
(3) Lecture Notes on Section 3.5.

2. Graphs of $f(x)$, $g'(x)$ and $h''(x)$ for $-5 \le x \le 5$ are given below.

a. Estimate **graphically** all possible inflection points c of $f(x)$ in the interval $(-5,\ 5)$. Determine interval(s) on which
(1) $f(x)$ is concave up and $f(x)$ is concave down. (2) $f(x)$ is increasing and concave down.



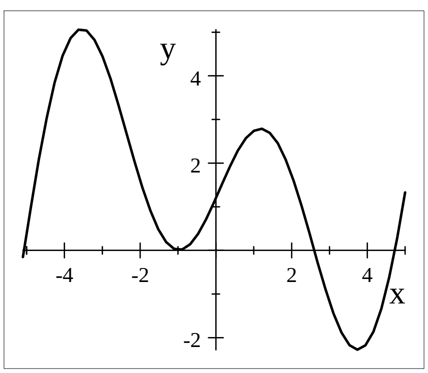
$y = f(x)$

Inflection points: $x = -2,\ 0,\ 2$

(1) $(-5, -2),\ (0, 2)$

(2) $(-5, -3.8),\ (0, 1)$

b. Estimate **graphically** all possible inflection points c of $g(x)$ in the interval $(-5,\ 5)$. Determine interval(s) on which
(1) $g(x)$ is concave up and $g(x)$ is concave down. (2) $g(x)$ is increasing and concave down.



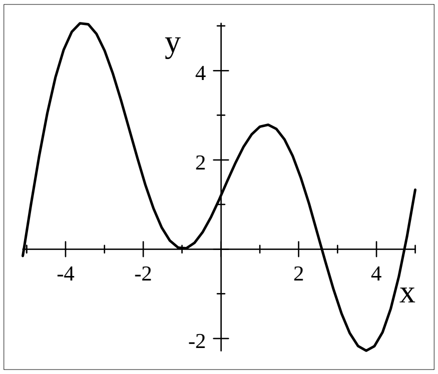
$y = g'(x)$

Inflection points: $x = -3.5,\ -1,\ 1,\ 3.5$

(1) $(-3.8, -1),\ (1,\ 3.8)$

(2) $(-3.8, -1),\ (1, 2.3)$

c. Estimate **graphically** all possible inflection points c of $h(x)$ in the interval $(-5,\ 5)$. Determine interval(s) on which
(1) $h(x)$ is concave up and $h(x)$ is concave down.
(2) If we also know $x = -4$, $x = -1$ and $x = 3$ are critical numbers of $h(x)$. Use the 2nd Derivative Test to classify each critical number as the location of a local maximum, local minimum or no conclusion.



$y = h''(x)$

Inflection points: $x = -5,\ 2.7,\ 4.7$

(1) concave up: $(-5,-1),\ (-1,2.5),\ (4.7,5)$
concave down: $\left(2.5,\ 4.7\right)$

(2) $h''(-4) > 0,$ so $h(-4)$ is a local minimum.
$h''(-1) = 0,$ so no conclusion from this test.
$h''(3) < 0,$ so $h(3)$ is a local maximum.

2. **Page 293**: Turn in the ones with *.

a. For each of the following problem, use Theorem 5.1 to determine algebraically intervals where $f(x)$ is concave up and concave down.
1-8: *2, *4, *8

For Problem 1-8, we first determine where $f''(x)$ is positive and where $f''(x)$ is negative.

2. $f(x) = x^4 - 6x^2 + 2x + 3,\ f'(x) = 4x^3 - 12x + 2,\ f''(x) = 12x^2 - 12$
Set $f''(x) = 0 : 12(x^2 - 1) = 12(x-1)(x+1) = 0, x = \pm 1$
Check signs of $f''(x)$ over $\left(-\infty,\ -1\right),\ (-1,1),\ (1,\infty)$:

$$f''(-2) = 12(-2-1)(-2+1) > 0$$
$$f''(0) = 12(-1)(1) < 0$$
$$f''(2) = 12(2-1)(2+1) > 0$$

	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
sign of $f''(x)$	+	−	+

Hence, $f(x)$ is concave up on $(-\infty,-1)$ and $(1,\infty)$ and concave down on $(-1,1)$.

4. $f(x) = x + 3(1-x)^{1/3},\ f'(x) = 1 - (1-x)^{-2/3},\ f''(x) = -\frac{2}{3}(1-x)^{-5/3} = -\dfrac{2}{3\sqrt[3]{(1-x)^5}}$

$D_f = (-\infty,\infty),\ D_{f''} = (-\infty,1) \cup (1,\infty)$
$f''(x) \neq 0$ and $f''(x)$ is not defined at $x = 1$. Check sign changes of $f''(x)$ on $\left(-\infty,\ 1\right)$ and $(1,\infty)$:

$$f''(0) = -\tfrac{2}{3} < 0$$
$$f''(2) = -\tfrac{2}{3}\,\frac{1}{\sqrt[3]{(1-2)^5}} > 0$$

	$\left(-\infty,\ 1\right)$	$(1,\infty)$
sign of $f''(x)$	−	+

$f(x)$ is concave up on $(1,\infty)$ and is concave down on $(-\infty,1)$.

8. $f(x) = xe^{-4x},\ f'(x) = e^{-4x} - 4xe^{-4x} = e^{-4x}(1-4x)$
$f''(x) = -4e^{-4x}(1-4x) - 4e^{-4x} = -4e^{-4x}(1-4x+1) = -4e^{-4x}(2-4x) = -8e^{-4x}(1-2x)$
$f''(x) = 0 : 1 - 2x = 0,\ x = \dfrac{1}{2}.$
Check sign changes of $f''(x)$ on $\left(-\infty,\ \dfrac{1}{2}\right),\ \left(\dfrac{1}{2},\ \infty\right)$:

$$f''(0) = -8(1)(1) < 0$$
$$f''(1) = -8e^{-4}(1-2) > 0$$

	$\left(-\infty,\ \frac{1}{2}\right)$	$\left(\frac{1}{2},\ \infty\right)$
sign of $f''(x)$	−	+

$f(x)$ is concave up on $\left(\dfrac{1}{2},\ \infty\right)$, and concave down on $\left(-\infty,\ \dfrac{1}{2}\right)$.

b. For each of the following problem, use Theorem 5.2 (the 2nd Derivative Test) to classify each critical number as the location of a local maximum, local minimum or no conclusion.
9-14: *10, 11, *12

10. $f(x) = x^4 + 4x^2 + 1,\ f'(x) = 4x^3 + 8x = 4x(x^2 + 2),\ f''(x) = 12x^2 + 8.$

(1) Critical numbers: $f'(x) = 0,\ x = 0.$

(2) Apply the 2nd Derivative Test: $f''(0) = 8 > 0,\ f(0) = 1$ is a local minimum value.

12. $f(x) = e^{-x^2},\ f'(x) = -2xe^{-x^2},\ f''(x) = -2\left(e^{-x^2} - 2x^2e^{-x^2}\right) = -2e^{-x^2}(1 - 2x^2).$

$D_f = (-\infty, \infty),\ D_{f'} = (-\infty, \infty),\ D_{f''} = (-\infty, \infty)$

(1) Critical numbers: $f'(x) = 0,\ x = 0$

(2) Apply the 2nd Derivative Test: $f''(0) = -2e^0(1 - 0) < 0,\ f(0) = 1$ is a local maximum value.

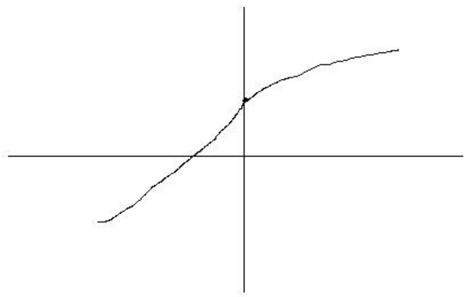
c. 41-44: 42*, extra points 44.

42. $f(0) = 2$

$f'(x) \geq 0$ for all x, $f'(0) = 1$

$f''(x) > 0$ for $x < 0$

$f''(x) < 0$ for $x > 0$



a.

4. **Page 284**: Turn in the ones with *.
35-38: 38*, extra points 36.

38.