

Math 131 - Spring, 2008 Homework 3 Due Tuesday, Jan 18 Name: _____

Mathematics Quote: *Don't just read it; fight it! Ask your own questions, look for your own examples, discover your own proofs. Is the hypothesis necessary? Is the converse true? What happens in the classical special case? What about the degenerate cases? Where does the proof use the hypothesis?*

- Halmos, Paul R. I Want to be a Mathematician, Washington: MAA Spectrum, 1985.

1. Reading assignment for Section 1.3:

- (1) Theorems 3.1-3.4.
- (2) Examples 3.1, 3.2, 3.3, 3.4 and 3.5.

2. Use Theorem 3.1-3.4 to find algebraically the following limits.

- a. (i) $\lim_{x \rightarrow -1} (2x^2 - x + 1)$ (ii) $\lim_{x \rightarrow 2} \sqrt{x^2 - 2}$
- b. (i) $\lim_{x \rightarrow 0} \frac{e^{-x/2}}{x^2 + \pi}$ (ii) $\lim_{x \rightarrow 2} \ln(x^2 - 3)$
- c. (i) $\lim_{x \rightarrow \pi/2} \cos^2(2x)$ (ii) $\lim_{x \rightarrow \pi/2} x \sin(3x)$

3. Determine first the type of each limit and then compute algebraically the limit if it exists:

- a. (i) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + 2}$ (ii) $\lim_{x \rightarrow 1} \frac{x^2 + 1}{x^2 - 1}$ (iii) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 1}$
- b. $\lim_{x \rightarrow 1} \frac{x^2 - 4}{x^2 + 5x + 6}$
- c. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 + 2x - 3}$
- d. $\lim_{h \rightarrow 0} \frac{(h + 1)^2 - 1}{h}$
- e. $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$
- f. $\lim_{h \rightarrow 0} \frac{\sqrt{h + 3} - \sqrt{3}}{h}$
- g. **Extra points:**
 - (i) $\lim_{h \rightarrow 0} \frac{(h + 1)^3 - 1}{h}$ (ii) $\lim_{x \rightarrow 0} \frac{2 - \sqrt{x + 4}}{3x}$ (iii) $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + x + 4} - 2}{x^2 + x}$

4. Use the fact that $\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{\theta} = 1$ to compute algebraically the limit if it exists.

- a. $\lim_{x \rightarrow 0} \frac{\sin(\pi x)}{x}$
- b. $\lim_{x \rightarrow 0} \frac{\tan(2x)}{x}$
- c. **Extra points:**
 - (i) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{5x}$ (ii) $\lim_{x \rightarrow 0} \frac{\sin^2(3x)}{x^2}$