

**Math 131 - Spring, 2008 Homework 4 Due Jan 22 Name: \_\_\_\_\_**

Mathematics Quote: *Don't just read it; fight it! Ask your own questions, look for your own examples, discover your own proofs. Is the hypothesis necessary? Is the converse true? What happens in the classical special case? What about the degenerate cases? Where does the proof use the hypothesis?*

- Halmos, Paul R. I Want to be a Mathematician, Washington: MAA Spectrum, 1985.

1. Let  $f(x) = \frac{x^2 - 4}{x - 2}$  and  $g(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$ . Are functions  $f(x)$  and  $g(x)$  the same? If your answer is yes, explain why. If your answer is no, specify the difference between  $f(x)$  and  $g(x)$ .  
No.  $f(2)$  is not defined and  $g(2) = 4$ .

2. Reading assignment for Section 1.4:  
(1) Definition 4.1.  
(2) Study Figures 1.22a-d on Page 98.

3. State the definition of the **continuity** of  $f(x)$  at  $x = a$ .

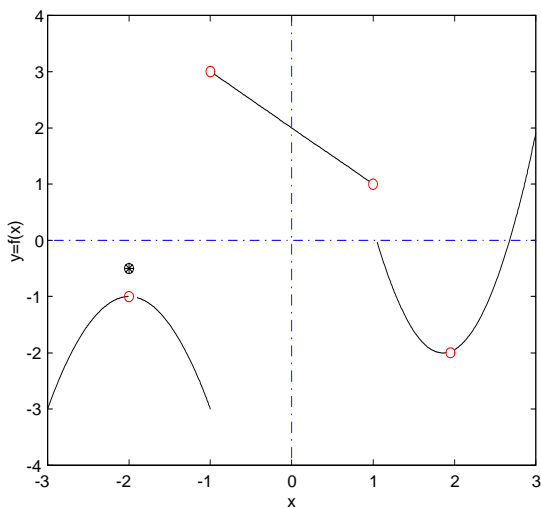
$f(x)$  is continuous at  $x = a$  if

- (i)  $f(a)$  is defined;
- (ii)  $\lim_{x \rightarrow a} f(x)$  exists; and
- (iii)  $\lim_{x \rightarrow a} f(x) = f(a)$

4. The graph of  $f(x)$  is given below. We know that  $f(x)$  is not continuous at

- (i)  $x = -2$  (ii)  $x = -1$  (iii)  $x = 1$  and (iv)  $x = 2$ .

For each of these 4 discontinuity points, specify a **continuity condition** (given in the definition of continuity) that fails to satisfy.



(i) $x = -2$
$\lim_{x \rightarrow -2} f(x) \neq f(-2)$
(ii) $x = -1$
$\lim_{x \rightarrow -1} f(x)$ DNE
(iii) $x = 1$
$\lim_{x \rightarrow 1} f(x)$ DNE
(iv) $x = 2$
$f(2)$ is not defined.

5. Consider the function  $f(x)$  whose graph is given in Problem 6 on Page 107. We know that  $f(x)$  is not continuous at

(i)  $x = -2$    (ii)  $x = 0$    and   (iii)  $x = 2$ .

For each of these 3 discontinuity points, specify a **continuity condition** (given in the definition of continuity) that fails to satisfy.

(i)  $f(x)$  is not continuous at  $x = -2$  because  $\lim_{x \rightarrow -2} f(x)$  DNE

(ii)  $f(x)$  is not continuous at  $x = 0$  because  $\lim_{x \rightarrow 0} f(x) \neq f(0)$ .

(iii)  $f(x)$  is not continuous at  $x = 2$  because  $\lim_{x \rightarrow 2} f(x)$  DNE