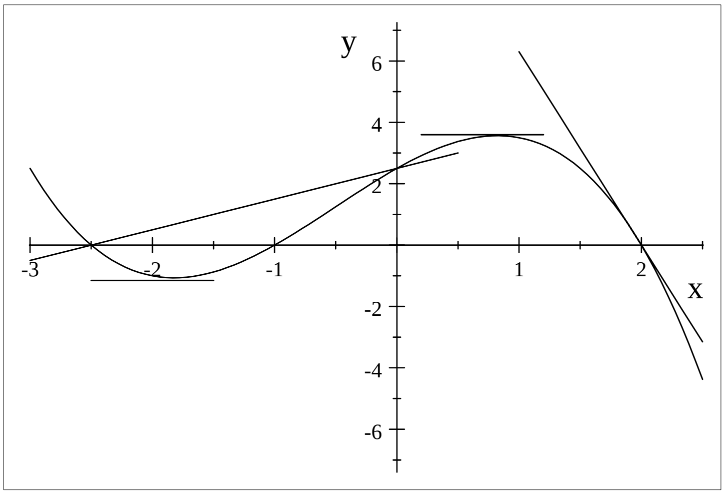
Math 131 Spring, 2008 Quiz 4 Name: ___Solutions_

Show your work in details and provide reasons to support your answers.

1. (3pts) The graph of $f(x)$ is given below.

Version (a):

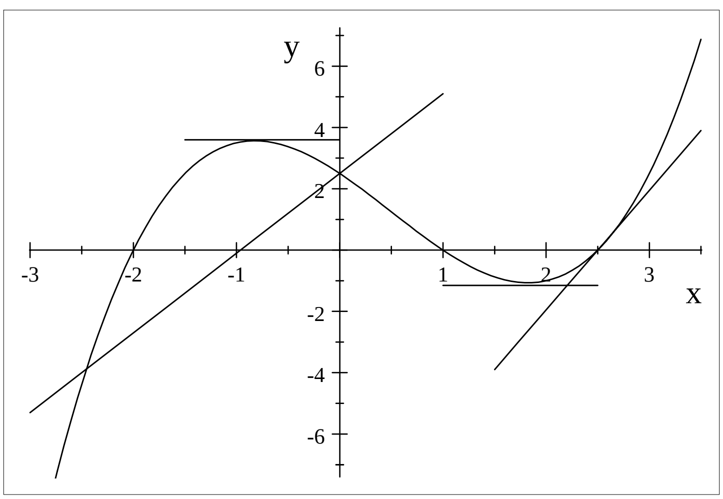
- **a**. Sketch **the secant line** that passes through points on the graph of $f(x)$ where $x = -2.5$ and $x = 0$.
- **b**. Sketch **the tangent line** to the graph of $f(x)$ at points where $x = 2$.
- **c**. Estimate all x values at which the slope of the tangent line to the curve $y = f(x)$ is 0.



c. Estimate x where the slope of
tangent line is 0 ($m = 0$):
$x \approx 0.8$, $x \approx -1.9$

Version (b):

- **a**. Sketch **the secant line** that passes through points on the graph of $f(x)$ where $x = -2.5$ and $x = 0$.
- **b**. Sketch **the tangent line** to the graph of $f(x)$ at points where $x = 2.5$.
- **c**. Estimate all x values at which the slope of the tangent line to the curve $y = f(x)$ is 0.



c. Estimate x where the slope of
tangent line is 0 ($m = 0$):
$x \approx 1.9$, $x \approx -0.9$

2. (5pts) Find the **equation** of the tangent line to the curve $y = f(x)$ at the point where $x = a$.

Version (a): $y = x^2 - 5x$, $a = 2$

(1) $f(x) = x^2 - 5x$, $f(2) = 2^2 - 5(2) = -6$

(2) $\dfrac{f(2+h) - f(2)}{h} = \dfrac{(2+h)^2 - 5(2+h) - (-6)}{h} = \dfrac{4 + 4h + h^2 - 10 - 5h + 6}{h}$

$= \dfrac{h^2 - h}{h} = \dfrac{h(h-1)}{h} = h - 1$

(3) $m = \lim_{h \to 0}(h - 1) = -1$

(4) the equation of the tangent line: $y - (-6) = (-3)(x - 2)$ or $y = -3x + 6 - 5 = -3x$

Version (b): $y = x^2 - 3x,\ a = -2.$

(1) $f(x) = x^2 - 3x,\ f(-2) = (-2)^2 - 3(-2) = 10$

(2) $\dfrac{f(-2+h) - f(-2)}{h} = \dfrac{(-2+h)^2 - 3(-2+h) - 10}{h} = \dfrac{4 - 4h + h^2 + 6 - 3h - 10}{h}$

$= \dfrac{h^2 - 7h}{h} = \dfrac{h(h-7)}{h} = h - 7$

(3) $m = \lim_{h\to 0}(h - 7) = -7$

(4) the equation of the tangent line: $y - 10 = (-7)(x+2)$ or $y = -7x - 14 + 10 = -7x - 4$

3. (4pts) Find the **slope** of the tangent line to the curve $y = f(x)$ at the point where $x = a$.

Version (a): $y = \dfrac{2}{x+1},\ a = 1.$

(1) $f(x) = \dfrac{2}{x+1},\ f(1) = \dfrac{2}{1+1} = 1$

(2)

$\dfrac{f(1+h) - f(1)}{h} = \dfrac{\dfrac{2}{(1+h)+1} - 1}{h} = \dfrac{1}{h}\left(\dfrac{2}{2+h} - 1\right) = \dfrac{1}{h}\left(\dfrac{2}{2+h} - \dfrac{2+h}{2+h}\right) = \dfrac{1}{h}\left(\dfrac{2-2-h}{2+h}\right)$

$= \dfrac{1}{h}\left(\dfrac{-h}{2+h}\right) = \dfrac{-1}{2+h}$

(3) $m = \lim_{h\to 0}\left(\dfrac{-1}{2+h}\right) = -\frac{1}{2}$

Version (b): $y = \sqrt{x+3},\ a = 1.$

(1) $f(x) = \sqrt{x+3},\ f(1) = \sqrt{1+3} = 2$

(2) $\dfrac{f(1+h) - f(1)}{h} = \dfrac{\sqrt{(1+h)+3} - 2}{h} = \dfrac{\sqrt{4+h} - 2}{h}\left(\dfrac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}\right) = \dfrac{4 + h - 4}{h\left(\sqrt{4+h} + 2\right)}$

$= \dfrac{h}{h\left(\sqrt{4+h} + 2\right)} = \dfrac{1}{\sqrt{4+h} + 2}$

(3) $m = \lim_{h\to 0}\dfrac{1}{\sqrt{4+h} + 2} = \dfrac{1}{4}$