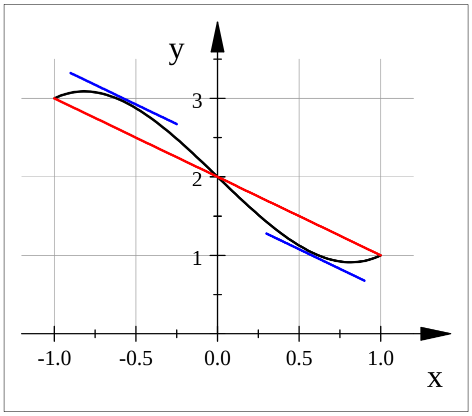
MATH 131-03, Fall, 20117 Exam 4 Name: ____________________

Please follow given instructions and show your work in detail.

1. (14pts)



(1) The graph of $f(x) = x^3 - 2x + 2$ for x in $[-1, 1]$ is given at the left. Use the graph of f to estimate all possible values of c in $(-1, 1)$ that satisfies the conclusion of the Mean-Value Theorem. Show your work graphically.

$c = -0.6$, $c = 0.6$

(2) Find all possible values of c algebraically.

$f(1) = 1$, $f(-1) = -1 + 2 + 2 = 3$

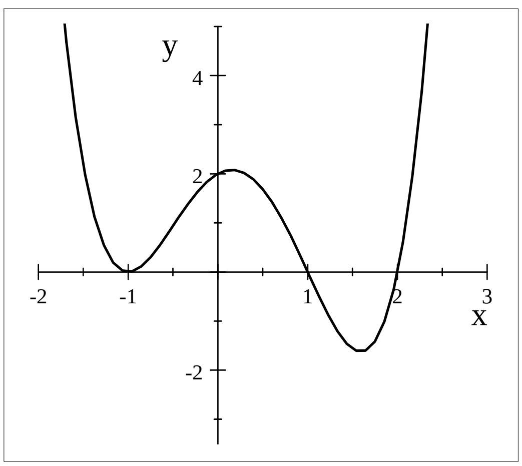
$m_{\sec} = \frac{1-3}{2} = -1$,

$f'(x) = 3x^2 - 2 = -1$, $3x^2 = 1$, $x = \pm\frac{1}{\sqrt{3}}$

2. (10pts) The graph of $f'(x)$ (derivative of $f(x)$) is given below.

(1) Find all critical numbers of $f(x)$.

$f'(x) = 0$: $x = -1$, $x = 1$, $x = 2$



$y = f'(x)$

(2) On what interval is f decreasing?

$f'(x) < 0$: $(1, 2)$

(3) At what value of x does f have a local maximum?

$x = c$ is a critical number and $f'(x)$ changes from + to − at $x = c$.

$x = 1$

(4) On what interval is f concave down?

$f'(x)$ is decreasing: $(-\infty, -1)$, $(0.2, 1.5)$

(5) What are the x-coordinates of the inflection points of f?

$f'(x)$ changes from increasing to decreasing or from decreasing to increasing: $x = -1$, $x = 0.2$, $x = 1.5$

3. (15pts) Let $f'(x) = 3(x-1)(x+2)$. Suppose we know the domain of $f(x)$ is $(-\infty, \infty)$.

(1) Find all critical numbers of $f(x)$.

$x = 1$, $x = -2$

(2) On what interval is f decreasing? Why? At what value of x does f have a local maximum?

	$(-\infty,-2)$	$(-2,1)$	$(1,\infty)$
$f'(x)$	$+$	$-$	$+$

$f'(x) < 0$ on $(-2,1)$ so $f(x)$ is decreasing on $(-2,1)$.

$f(-2)$ is a local maximum since $f(x)$ changes from increasing to decreasing.

(3) Compute $f''(x)$

$f''(x) = 3((x+2)+(x-1)) = 3(2x+1)$

(4) Determine the interval(s) on which $f(x)$ is concave up. Give the x-coordinate of each inflection point.

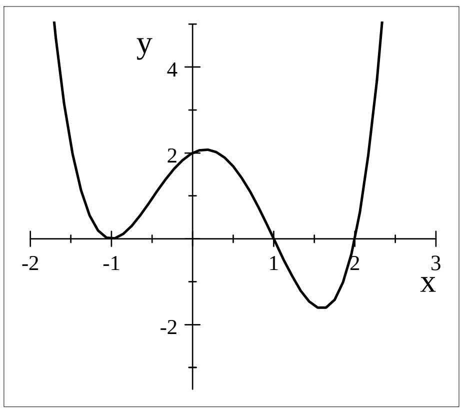
	$(-\infty,-\frac{1}{2})$	$(-\frac{1}{2},\infty)$
$f'(x)$	$-$	$+$

$f''(x) > 0$ on $(-\frac{1}{2},\infty)$ so $f(x)$ is concave up on $(-\frac{1}{2},\infty)$.

(5) Determine the interval(s) on which $f(x)$ is decreasing and concave up.

$f'(x) < 0$ and $f''(x) > 0$ on $(-\frac{1}{2},1)$, $f(x)$ is decreasing and concave up on $(-\frac{1}{2},1)$.

4. (8pts) The graph of $f''(x)$ (the second derivative of $f(x)$) is given below.



$y = f''(x)$

(1) On what interval is f concave up? Why?

$f'' > 0$: $(-\infty,-1)$, $(-1,1)$, $(2,\infty)$

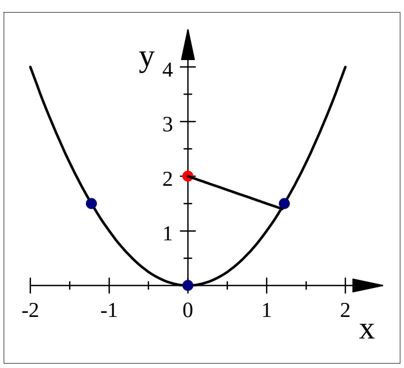
(2) What is the x-coordinate of each inflection point of f?

f'' changes its signs: $x = 1$, $x = 2$

(3) If we also know that $f'(0) = 0$, determine if $f(0)$ is a local maximum, a local minimum or neither. Why?

$f''(0) > 0$: $f(0)$ is a local minimum.

5. (10pts) Find algebraically the point on the curve $y = x^2$ closest to the point $(0,2)$. Show your work in detail.



$$D(x) = \sqrt{(0-x)^2 + (2-y)^2}$$
$$= \sqrt{x^2 + (2-x^2)^2}$$

$$D'(x) = \frac{1}{2\sqrt{x^2+(2-x^2)^2}}(2x + 2(2-x^2)(-2x)) = \frac{1}{2\sqrt{x^2+(2-x^2)^2}}2x(1-4+2x^3)$$
$$= \frac{1}{2\sqrt{x^2+(2-x^2)^2}}2x(-3+2x^3)$$

$D'(x) = 0$, $x = 0$ or $-3+2x^3 = 0$, that is $x = \pm\sqrt{\frac{3}{2}}$, $y = \frac{3}{2}$

6. (10pts) Find the general antiderivative $F(x)$ of $f(x)$.

(1) $f(x) = 2\sec(x)\tan(x) - 3\sec^2(x) + \dfrac{1}{1+x^2}$

$F(x) = 2\sec(x) - 3\tan(x) + \tan^{-1}(x) + C$

(2) $f(x) = \dfrac{x - 3 + \sqrt{x}}{x^2} = \dfrac{1}{x} - 3x^{-2} + x^{-3/2}$

$F(x) = \ln|x| + 3x^{-1} - 2x^{-1/2} + C$

7. (5pts) Find $f(t)$ if we know the $f'(t) = 3e^t + 2\sin(t)$ and $f(0) = 2$.

$f(t) = 3e^t - 2\cos(t) + C, \; f(0) = 3 - 2 + C = 2, \; C = 1$

$f(t) = 3e^t - 2\cos(t) + 1$

8. (4pts) Express the limit $\lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{i=1}^{n}\sqrt{\left(1+i\frac{2}{n}\right)^3 + \left(1+i\frac{2}{n}\right)}$ as $\int_a^b f(x)dx$.

$f(x) = \sqrt{x^3 + x}, \; b - a = 2, \; a = 1, \; b = 2 + 1 = 3$

$\int_a^b f(x)dx = \int_1^3 \sqrt{x^3 + x}\, dx$

9. (24pts) The graph of $f(x) = 3x - x^2$ for x in $[0,2]$ is given below.

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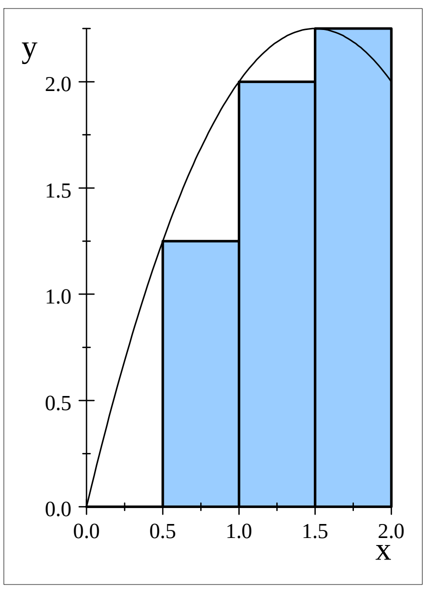

(1) Sketch and shade L_4.

(2) Estimate the area under the graph of $f(x) = 3x - x^2$ for x in $[0,2]$ above the x-axis using M_6.

$\triangle x = \frac{2}{6} = \frac{1}{3}, \; x_i = i\frac{1}{3}, \; \bar{x}_i = i\frac{1}{3} - \frac{1}{2}\left(\frac{1}{3}\right)$

$M_6 = \left(\frac{1}{3}\right)\sum_{i=1}^{6}\left(3\left(i\frac{1}{3} - \frac{1}{6}\right) - \left(i\frac{1}{3} - \frac{1}{6}\right)^2\right)$

$= \frac{181}{54} = 3.3519$

(3) Evaluate $\int_0^2 (3x - x^2)dx$ using the Fundamental Theorem of Calculus.

$\int_0^2 (3x - x^2)dx = \frac{10}{3}$

(4) Evaluate $\int_0^2 (3x - x^2)dx$ by the definition of definite integral. ($\sum_{i=1}^{n} 1 = n, \; \sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$)

$\int_0^2 (3x - x^2)dx = \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(3(i\frac{2}{n}) - (i\frac{2}{n})^2\right) = \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(i(\frac{6}{n}) - i^2\frac{4}{n^2}\right)$

$= \lim_{n\to\infty}\left[\frac{12}{n^2}\sum_{i=1}^{n} i - \frac{8}{n^3}\sum_{i=1}^{n} i^2\right] = \lim_{n\to\infty}\left(\frac{12}{n^2}\right)\frac{n(n+1)}{2} - \frac{8}{n^3}\frac{n(n+1)(2n+1)}{6}$