MATH 131-03, Fall, 2017 Homework Section 2.7 - 1 - Solutions

Page 148: 4(a)(ii), 10, 14

4(a)(ii) $y = x - x^3,\ (1,0)$

$$m_{\tan} = \lim_{h\to 0}\frac{y(1+h)-y(1)}{h} = \lim_{h\to 0}\frac{(1+h)-(1+h)^3-0}{h}$$
$$= \lim_{h\to 0}\frac{(1+h)-(1+3h+3h^2+h^3)}{h} = \lim_{h\to 0}\frac{-2h-3h^2-h^3}{h}$$
$$= \lim_{h\to 0} -2-3h-h^2 = -2$$

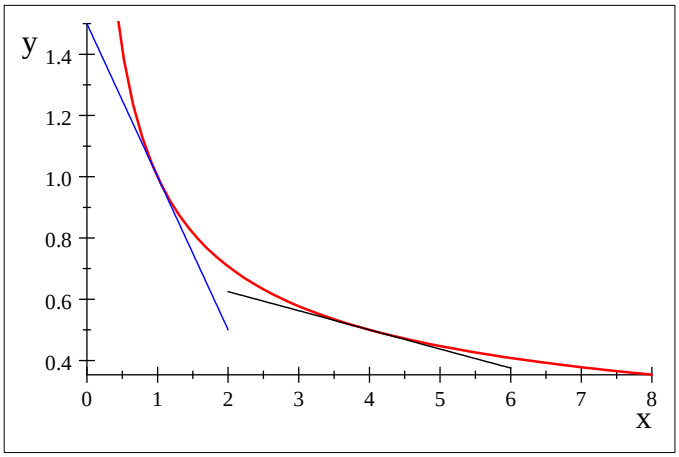
10. $y = \dfrac{1}{\sqrt{x}}$

(a) $$\frac{y(a+h)-y(a)}{h} = \frac{1/\sqrt{a+h}-1/\sqrt{a}}{h} = \frac{1}{h}\left[\frac{\sqrt{a}-\sqrt{a+h}}{\sqrt{a+h}\sqrt{a}}\right]\frac{\left(\sqrt{a}+\sqrt{a+h}\right)}{\left(\sqrt{a}+\sqrt{a+h}\right)}$$
$$= \frac{1}{h}\left[\frac{a-(a+h)}{\sqrt{a+h}\sqrt{a}\left(\sqrt{a}+\sqrt{a+h}\right)}\right] = \frac{1}{h}\left[\frac{-h}{\sqrt{a+h}\sqrt{a}\left(\sqrt{a}+\sqrt{a+h}\right)}\right]$$
$$= \frac{-1}{\sqrt{a+h}\sqrt{a}\left(\sqrt{a}+\sqrt{a+h}\right)}$$
$$m_{\tan} = \lim_{h\to 0}\frac{-1}{\sqrt{a+h}\sqrt{a}\left(\sqrt{a}+\sqrt{a+h}\right)} = \frac{-1}{\sqrt{a}\sqrt{a}\left(\sqrt{a}+\sqrt{a}\right)} = \frac{-1}{2\left(\sqrt{a}\right)^3}$$

(b) At $(1,1)$, $m_{\tan} = \dfrac{-1}{2}$ and the equation of the tangent line: $y - 1 = -\frac{1}{2}(x-1)$

At $(4, \frac{1}{2})$, $m_{\tan} = \dfrac{-1}{2\left(\sqrt{4}\right)^3} = -\dfrac{1}{16}$ and the equation of the tangent line: $y - \frac{1}{2} = -\frac{1}{16}(x-4)$

(c)



14. $H(t) = 10t - 1.86t^2$ in meters, t in seconds

(a) $$v(1) = \lim_{h\to 0}\frac{H(1+h)-H(1)}{h} = \lim_{h\to 0}\frac{10(1+h)-1.86(1+h)^2-\left(10(1)-1.86(1)^2\right)}{h}$$
$$= \lim_{h\to 0}\frac{10(1+h)-1.86(1+2h+h^2)-(10-1.86)}{h} = \lim_{h\to 0}\frac{10h-3.72h-1.86h^2}{h}$$
$$= \lim_{h\to 0}(6.28-1.86h) = 6.28 \text{ m/s}$$

(b) $$v(a) = \lim_{h\to 0}\frac{H(a+h)-H(a)}{h} = \lim_{h\to 0}\frac{10(a+h)-1.86(a+h)^2-\left(10(a)-1.86(a)^2\right)}{h}$$
$$= \lim_{h\to 0}\frac{10a+10h-1.86a^2-3.72ah-1.86h^2-10a+1.86a^2}{h}$$
$$= \lim_{h\to 0}\frac{10h-3.72ah-1.86h^2}{h}$$

$= \lim_{h\to 0}(10 - 3.72a - 1.86h) = 10 - 3.72a$ m/s

(c) The rock hit the surface if $H(t) = 0$:

$H(t) = 10t - 1.86t^2 = t(10 - 1.86t) = 0, \quad t = \frac{10}{1.86} = 5.376$ seconds.

(d) $v\left(\frac{10}{1.86}\right) = 10 - 3.72\left(\frac{10}{1.86}\right) = -10.0$