MATH 131-03, Fall, 2017 Homework Section 4.1.2 - Solutions

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44. $f(x) = \frac{1}{x^2}\ln(x)$

(1) Domain of $f(x)$: $x > 0$

(2) $f'(x) = -2x^{-3}\ln(x) + x^{-2}\left(\frac{1}{x}\right) = \frac{-1}{x^3}(-2\ln(x) + 1)$

(3) Set $f'(x) = 0$, $-2\ln(x) + 1 = 0$, $\ln(x) = \frac{1}{2}$, $x = e^{1/2}$

Critical number of type 1: $f'(x) = 0$, $x = e^{1/2}$

Critical number of type 2: $f'(x)$ is not defined at $x = 0$

but it is not in the domain of f so there is no critical number of type 2.

60. $f(x) = xe^{x/2}$, $[-3, 1]$

(i) Find all critical numbers of $f(x)$ in $[-3, 1]$

(1) $f'(x) = e^{x/2} + \frac{1}{2}xe^{x/2} = e^{x/2}(1 + \frac{1}{2}x)$

(2) $f'(x) = 0$, $1 + \frac{1}{2}x = 0$, $x = -2$

(ii) $f(-3) = -3e^{-3/2} = -0.669\,390\,48$, $f(1) = e^{1/2} = 1.648\,721\,27$, $f(-2) = -2e^{-1} = -0.735\,758\,882$

the absolute max value is $1.648\,721\,27$

the absolute min value is $-0.735\,758\,882$

62. $f(x) = x - 2\tan^{-1}(x)$, $[0, 4]$

(i) Find all critical numbers of $f(x)$ in $[0, 4]$

(1) $f'(x) = 1 - 2\left(\frac{1}{1+x^2}\right) = \frac{1+x^2-2}{1+x^2} = \frac{x^2-1}{1+x^2}$

(2) $f'(x) = 0$, $x^2 - 1 = 0$, $x = \pm 1$, but only $x = 1$ is in $[0, 4]$, critical number: $x = 1$

(ii) $f(0) = 0$, $f(4) = 4 - 2\tan^{-1}(4) = 1.348\,364\,67$, $f(1) = 1 - 2\tan^{-1}(1) = -0.570\,796\,327$

the absolute max value is $1.348\,364\,67$

the absolute min value is $-0.570\,796\,327$