MATH 131-03, Fall, 2017 Homework Section 4.7 - Solutions

Page 336: 2, 8, 14, 18, 34

2. Know $x - y = 100,\ x = 100 + y$

Product $P = xy = (100 + y)y$, min $P(y)$ for y in $(-\infty, \infty)$

(1) critical number of P:

$P'(y) = 100 + 2y,\ P'(y) = 0,\ y = -\frac{100}{2} = -50$

(2) Check P at $-\infty,\ -50$ and ∞ :

$P(-\infty) = \infty,\ P(\infty) = \infty,\ P(-50) = 50(-50) = -2500$ minimum

$x = 100 + (-50) = 50$

Ans: $x = 50$ and $y = -50$

8. Know area $= LW = 1000,\ L = \frac{1000}{W}$

Perimeter $P = 2L + 2W$, min $P = 2(\frac{1000}{W}) + 2W$ for W in $(0, \infty)$

(1) Critical number of P:

$P'(W) = 2\left(-\frac{1000}{W^2} + 1\right) = 0,\ W = \sqrt{1000}$

(2) Check P at 0, $\sqrt{1000}$ and ∞

$P(0) = \infty,\ P(\infty) = \infty,\ P\left(\sqrt{1000}\right) = 2\left(\frac{1000}{\sqrt{1000}} + \sqrt{1000}\right) = 4\sqrt{1000}$ minimum

Ans: $W = \sqrt{1000}$ and $L = \frac{1000}{\sqrt{1000}} = \sqrt{1000}$

14 Volume of the box $V = x^2h = 32000,\ h = \dfrac{32000}{x^2}$

Surface area $S = x^2 + 4hx = x^2 + \frac{4(32000)}{x}$, min S for x in $(0, \infty)$

(1) Critical number of S:

$S'(x) = 2x - \dfrac{128000}{x^2} = \dfrac{2x^3 - 128000}{x^2} = 0,$

$2(x^3 - 64000) = 0,\ x = \sqrt[3]{64000},\ h = \dfrac{32000}{\sqrt[2]{64000}} = 126.49$

(2) Check S at 0, $\sqrt[3]{64000}$, ∞

$S(0) = \infty,\ S(\infty) = \infty,\ S\left(\sqrt[3]{64000}\right) = (64000)^{2/3} + \frac{4(32000)}{\sqrt[3]{64000}} = 4800.0$ minimum

18. Know $C = \frac{1}{2}(20x) + 20y + 20x = 30x + 20y = 5000,\ y = \frac{1}{20}(5000 - 30x)$

Area $= A = xy = \frac{x}{20}(5000 - 30x)$ min $A(x)$ for x in $\left(0, \frac{5000}{30}\right)$

(1) Critical number of A:

$A'(x) = \frac{1}{20}(5000 - 60x) = 0,\ x = \frac{5000}{60} = \frac{250}{3}, y = \frac{1}{20}\left(5000 - 30\left(\frac{250}{3}\right)\right) = 125$

(2) Check A at 0, $\frac{250}{3}$, $\frac{5000}{30}$:

$A(0) = 0,\ A\left(\frac{5000}{30}\right) = 0$

$A\left(\frac{250}{3}\right) = \frac{1}{20}\left(\frac{250}{3}\right)\left(5000 - 30\left(\frac{250}{3}\right)\right) = \frac{31250}{3} = 10416.6667$

34. Perimeter $P = 2r + 2h + \frac{1}{2}(2\pi r) = 2r + 2h + \pi r = 30,\ h = 15 - \dfrac{(2+\pi)}{2}r$

Area $A = 2rh + \frac{1}{2}\pi r^2 = A(r) = 2r\left(15 - \dfrac{(2+\pi)}{2}r\right) + \frac{1}{2}\pi r^2$, max A for r in $\left(0, \frac{30}{2+\pi}\right)$

(1) $A'(r) = 2(15 - (2+\pi)r) + \pi r = 0,$

$30 - (4 + 2\pi)r + \pi r = 30 - (4+\pi)r = 0,\ r = \dfrac{30}{4+\pi} = 4.20074365$

$h = 15 - \dfrac{(2+\pi)}{2}\dfrac{30}{4+\pi} = 4.20074365$

(2) Check A at 0, $\frac{30}{4+\pi}$, $\frac{30}{2+\pi}$

$A(0) = 0$, $A\left(\frac{30}{2+\pi}\right) = 53.477002$, $A\left(\frac{30}{4+\pi}\right) = 98.303649$ maximum