MATH 131 Fall, 2017 Quiz 10 Name: __Solutions_______

No Calculator. Show your work in details.

1. (6pts) Find the derivative of each function. It is not required to simply the answers.

(1) $h(x) = e^{-2x}\cos(3x)$

$$h(x) = (-2e^{-2x})\cos(3x) + e^{-2x}(-3\sin(3x))$$
$$= -2e^{-2x}\cos(3x) - 3e^{-2x}\sin(3x)$$

(2) $h(x) = \sqrt{\dfrac{1+\sin(2x)}{1-\cos(2x)}} = \left(\dfrac{1+\sin(2x)}{1-\cos(2x)}\right)^{1/2}$

$$h'(x) = \tfrac{1}{2}\left(\frac{1+\sin(2x)}{1-\cos(2x)}\right)^{-1/2}\frac{d}{dx}\left(\frac{1+\sin(2x)}{1-\cos(2x)}\right)$$
$$= \tfrac{1}{2}\left(\frac{1+\sin(2x)}{1-\cos(2x)}\right)^{-1/2}\left(\frac{(0+2\cos(2x))(1-\cos(2x)) - (1+\sin(2x))(0-(-2\sin(2x)))}{(1-\cos(2x))^2}\right)$$
$$= \tfrac{1}{2}\left(\frac{1+\sin(2x)}{1-\cos(2x)}\right)^{-1/2}\left(\frac{(2\cos(2x))(1-\cos(2x)) - (1+\sin(2x))(2\sin(2x))}{(1-\cos(2x))^2}\right)$$

(3) $h(x) = \dfrac{(x^2+x)^5}{\sqrt{2x+3}}$ (using the Product Rule)

$$h(x) = (x^2+x)^5(2x+3)^{-1/2}$$
$$h'(x) = 5(x^2+x)^4(2x+3)^{-1/2} + (x^2+x)^5\left(-\tfrac{1}{2}(2x+3)^{-3/2}(2)\right)$$
$$= 5(x^2+x)^4(2x+3)^{-1/2} - (x^2+x)^5(2x+3)^{-3/2}$$

2. (2pts) Find $h''(x)$ where $h(x) = \tan(x^2)$.

$$h'(x) = \sec^2(x^2)(2x) = 2x\sec^2(x^2)$$
$$h''(x) = 2((1)\sec^2(x^2) + x(2\sec(x^2)(\sec(x^2)\tan(x^2)(2x))))$$
$$= 2(\sec^2(x^2) + 4x^2\sec^2(x^2)\tan(x^2))$$

3. (2pts) Let $h(x) = \sqrt{x^2 + 2f(x)}$. Find the equation of the tangent line to $y = h(x)$ at the point where $x = 1$ if we know $f(1) = 4$ and $f'(1) = -2$.

$$h(x) = (x^2 + 2f(x))^{1/2}$$
$$h(1) = \sqrt{1+2(4)} = 3$$
$$h'(x) = \tfrac{1}{2}(x^2+2f(x))^{-1/2}(2x + 2f'(x))$$
$$m = h'(1) = \tfrac{1}{2}(1^2 + 2f(1))^{-1/2}(2(1) + 2f'(1)) = \tfrac{1}{2}(1+2(4))^{-1/2}(2+2(-2))$$
$$= \tfrac{1}{2}\tfrac{-2}{3} = -\tfrac{1}{3}$$

equation of tangent line: $y - 3 = -\tfrac{1}{3}(x-1)$