MATH 131 Fall, 2017 Quiz 11 Name: _Solutions________

No Calculator. **Show your work in details**.

1. (3pts) Let $h(x) = \sqrt{4 - \ln(x)}$.

(1) Find the domain of $h(x)$.

$4 - \ln(x) \geq 0, \quad \ln(x) \leq 4, \quad x = e^{\ln(x)} \leq e^4$

Because the domain of $\ln(x)$ is $x > 0$, the domain of $h(x)$: $(0, e^4]$

(2) Compute $h'(x)$ and the domain of $h'(x)$.

$$h'(x) = \frac{d}{dx}\left[(4 - \ln(x))^{1/2}\right] = \frac{1}{2}(4 - \ln(x))^{-1/2}\left(0 - \frac{1}{x}\right) = \frac{-1}{2x\sqrt{4 - \ln(x)}}$$

the domain: $(0, e^4)$

(3) Give the equation of the tangent line to the curve $y = \sqrt{4 - \ln(x)}$ at the point where $x = 1$.

$$m = h'(1) = \frac{-1}{2(1)\sqrt{4 - \ln(1)}} = \frac{-1}{4}$$

$$h(1) = \sqrt{4 - \ln(1)} = 2$$

equation of the tangent line: $y - 2 = \frac{-1}{4}(x - 1)$

2. (7pts) Find the derivative of $h'(x)$. It is not required to simply the answers.

(1) $h(x) = \ln(\sqrt{x} + \cos(\pi x))$

$$h'(x) = \frac{1}{\sqrt{x} + \cos(\pi x)} \frac{d}{dx}[x^{1/2} + \cos(\pi x)] = \frac{1}{\sqrt{x} + \cos(\pi x)}\left[\tfrac{1}{2}x^{-1/2} - \pi\sin(\pi x)\right]$$

(2) $h(x) = \dfrac{(x^4 + e^{2x})^5}{(x + \sin(3x))^7}$. Use the Logarithmic Differentiation.

(i) $\ln(h(x)) = \ln\left(\dfrac{(x^4 + e^{2x})^5}{(x + \sin(3x))^7}\right) = 5\ln(x^4 + e^{2x}) - 7\ln(x + \sin(3x))$

(ii) $\dfrac{1}{h(x)}h'(x) = 5\dfrac{4x^3 + 2e^{2x}}{x^4 + e^{2x}} - 7\dfrac{1 - 3\cos(3x)}{x + \sin(3x)}$

(iii) $h'(x) = \dfrac{(x^4 + e^{2x})^5}{(x + \sin(3x))^7}\left[5\dfrac{4x^3 + 2e^{2x}}{x^4 + e^{2x}} - 7\dfrac{1 - 3\cos(3x)}{x + \sin(3x)}\right]$

(3) $h(x) = [\ln(x)]^{\tan(2x)}$. Use the Logarithmic Differentiation.

(i) $\ln(h(x)) = \tan(2x)\ln(\ln(x))$

(ii) $\dfrac{1}{h(x)}h'(x) = 2\sec^2(2x)\ln(\ln(x)) + \tan(2x)\dfrac{1}{\ln(x)}\left(\dfrac{1}{x}\right)$

(iii) $h'(x) = [\ln(x)]^{\tan(2x)}\left[2\sec^2(2x)\ln(\ln(x)) + \tan(2x)\dfrac{1}{\ln(x)}\left(\dfrac{1}{x}\right)\right]$