

No Calculator. Show your work in details.

1. (5pts) Compute  $h'(x)$  using the Logarithmic Differentiation.

$$(1) h(x) = \frac{(x - \ln(x))^5 (x^2 + \sin(2x))^{10}}{\sqrt[5]{e^{-2x} + x}}$$

$$\ln(h(x)) = \ln\left(\frac{(x - \ln(x))^5 (x^2 + \sin(2x))^{10}}{\sqrt[5]{e^{-2x} + x}}\right) = 5 \ln(x - \ln(x)) + 10 \ln(x^2 + \sin(2x)) - \frac{1}{5} \ln(e^{-2x} + x)$$

$$\frac{1}{h(x)} h'(x) = 5 \frac{1 - \frac{1}{x}}{x - \ln(x)} + 10 \frac{2x + 2 \cos(2x)}{x^2 + \sin(2x)} - \frac{1}{5} \frac{-2e^{-2x} + 1}{e^{-2x} + x}$$

$$h'(x) = \frac{(x - \ln(x))^5 (x^2 + \sin(2x))^{10}}{\sqrt[5]{e^{-2x} + x}} \left[ 5 \frac{1 - \frac{1}{x}}{x - \ln(x)} + 10 \frac{2x + 2 \cos(2x)}{x^2 + \sin(2x)} - \frac{1}{5} \frac{-2e^{-2x} + 1}{e^{-2x} + x} \right]$$

$$(2) h(x) = [\tan(\pi x)]^{\ln(x)}$$

$$\ln(h(x)) = \ln(x) \ln(\tan(\pi x))$$

$$\frac{1}{h(x)} h'(x) = \frac{1}{x} \ln(\tan(\pi x)) + \ln(x) \frac{1}{\tan(\pi x)} (\pi \sec^2(\pi x))$$

$$h'(x) = [\tan(\pi x)]^{\ln(x)} \left[ \frac{1}{x} \ln(\tan(\pi x)) + \ln(x) \frac{1}{\tan(\pi x)} (\pi \sec^2(\pi x)) \right]$$

2. (3pts) A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420.

- (1) Find an expression for the number of bacteria after  $t$  hours.

$$y = 100e^{kt}, y(1) = 100e^{k(1)} = 420, e^k = 4.2, k = \ln(4.2)$$

$$y = 100e^{\ln(4.2)t}$$

- (2) Find the number of bacteria after 3 hours.

$$y(3) = 100e^{\ln(4.2)(3)} = 1764.0$$

- (3) When will the population reach 10000?

$$10000 = 100e^{\ln(4.2)t}, e^{\ln(4.2)t} = 100, \ln(4.2)t = \ln(100)$$

$$t = \frac{\ln(100)}{\ln(4.2)} = 3.20898881 \text{ hours}$$

3. (2pts) When a cold drink is taken from a refrigerator, its temperature is  $5^\circ C$ . After 30 minutes in a room with temperature  $25^\circ C$ , the temperature of the drink has increased to  $20^\circ C$ . What is the temperature of the drink after 40 minutes?

$$T_s = 25, T_0 = 5, y_0 = 5 - 25 = -20, T(30) = 20, y(30) = 20 - 25 = -5$$

$$y = -20e^{kt}, y(30) = -20e^{k(30)} = -5, e^{30k} = \frac{-5}{-20} = 0.25,$$

$$k = \frac{1}{30} \ln(0.25) =$$

4.  $-0.046209812$

$$y(40) = -20e^{40/30 \ln(0.25)} = -3.14980262$$

$$T(40) = y(40) + 25 = -3.14980262 + 25 = 21.8501974$$