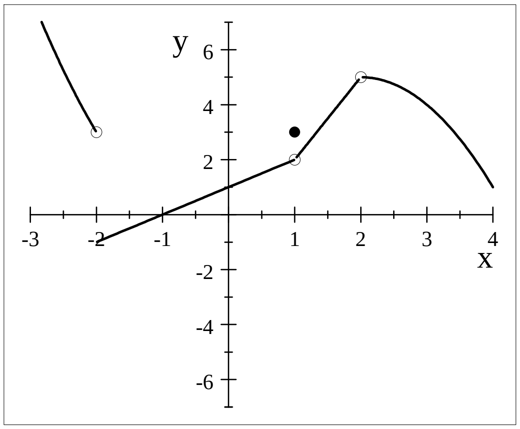
MATH 131 Fall, 2017 Quiz 4 Name: ____Solutions______

No Calculator. Show your work in details.

1. (3pt) Function $f(x)$ is said to be continuous at $x = a$ if and only if $\lim_{x\to a} f(x) = f(a)$. The graph of $f(x)$ is given below. We know $f(x)$ is not continuous at $x = -2$, $x = 1$ and $x = 2$. For each of these discontinuous points, explain by the **definition of continuity** why $f(x)$ is not continuous .



(1) $x = -2$

$\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$

or $\lim_{x\to -2} f(x)$ DNE

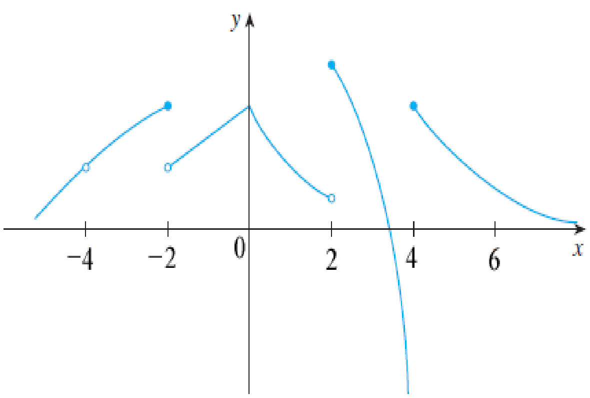
(2) $x = 1$

$\lim_{x\to 1} f(x) = 2$ but $f(2) = 3$

(3) $x = 2$

$f(2)$ is not defined.

2. (2pts) The graph of $f(x)$ is given below. We know $f(x)$ is not continuous at $x = -4$, $x = -2$, $x = 2$ and $x = 4$. Determine whether f is continuous from the right, or from the left, or neither at the given points:



(1) $x = -4$

Since $f(-4)$ is not defined, $f(x)$ is not continuous from the right, nor from the left at $x = -4$.

(2) $x = -2$

$f(x)$ is continuous at $x = -2$ from the left.

(3) $x = 2$

$f(x)$ is continuous at $x = 2$ from the right.

(4) $x = 4$

$f(x)$ is continuous at $x = 2$ from the right.

3. (3pts) Let $f(x) = \dfrac{x^2 - x - 2}{x^2 - 4}$.

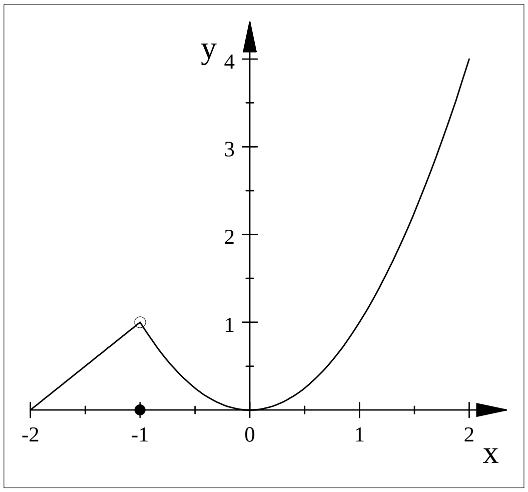
(1) Compute **algebraically** $\lim_{x\to 2} \dfrac{x^2 - x - 2}{x^2 - 4}$ if it exists.

$$\lim_{x\to 2} \frac{x^2 - x - 2}{x^2 - 4} \overset{\frac{0}{0}}{=} \lim_{x\to 2} \frac{(x-2)(x+1)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{x+1}{x+2} = \frac{3}{4}$$

(2) Is it possible to remove the discontinuity of $f(x)$ at $x = 2$? In other words, how would you define $f(2)$ in order to make f continuous at 2?

Yes, define $f(2) = \dfrac{3}{4}$ so $\lim_{x\to 2} \dfrac{x^2 - x - 2}{x^2 - 4} = f(2)$.

4. (2pts) Sketch the graph of $f(x) = \begin{cases} 2 + x \text{ if } x < -1 \\ 0 \text{ if } x = -1 \\ x^2 \text{ if } x > -1 \end{cases}$ for $-2 \le x \le 2$. Label your scales.



Find the x-value in $(-2,2)$ at which f is discontinuous and why.

$f(x)$ is not continuous at $x = -1$ because $f(-1) = 0$ but $\lim_{x\to -1} f(x) = 1$.