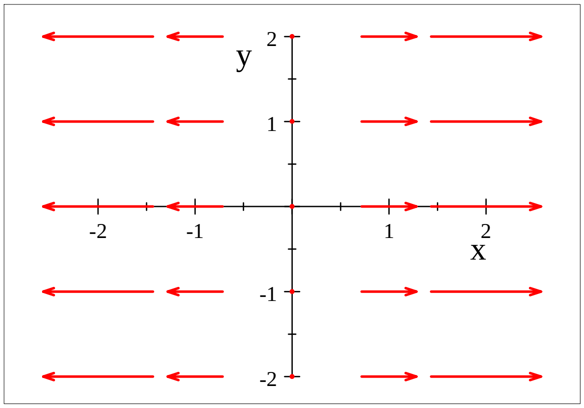
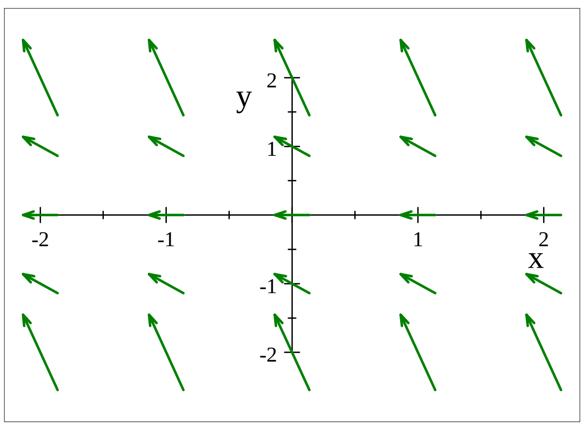
Homework Assignment 21 - (14.1) - Solutions

Page 1127- turn in the problems with (*) - April 8
1-10: 4*, 6*, x=-2,-1,0,1,2, y=-2,-1,0,1,2
11
13-18, 14*
23-34: 28*, 30*, 34*
35-42: 38*, 40*
Extra points: 22, 48, 52

4. $\vec{F}(x,y) = < 2x, 0 >$



6. $\vec{F}(x,y) = < -1, y^2 >$



10. $\vec{F}(x,y) = < -1,\ y^2 >$

14. $f(x,y) = x^2 - y^2$

$$\vec{F}(x,y) = \nabla f(x,y) = < 2x,\ -2y >$$

28. $\vec{F}(x,y) = \langle x^2 - y,\ x - y \rangle$, Determine if it is possible to fine $f(x,y)$ such that $\nabla f = \vec{F}$.

$$f_x = x^2 - y,\ f(x,y) = \int (x^2 - y)dx = \frac{1}{3}x^3 - yx + C(y)$$

$$f_y = -x + C'(y) = x - y,\ \ C'(y) = -y,\ \ C(y) = -\frac{1}{2}y^2 + C$$

$$f(x,y) = \frac{1}{3}x^3 - yx - \frac{1}{2}y^2 + C$$

Hence, $\vec{F}(x,y)$ is conservative.

30. $\vec{F}(x,y) =< y\cos x,\ \ \sin x - y >$. Determine if it is possible to fine $f(x,y)$ such that $\nabla f = \vec{F}$.

$$f_x = y\cos x, \quad f(x,y) = \int y\cos x dx = y\sin x + C(y)$$

$$f_y = \frac{\partial}{\partial y}(y\sin x + C(y)) = \sin x + C'(y) = \sin x - y, \quad C'(y) = -y, \quad C(y) = \int -y dy = -\frac{1}{2}y^2 + C$$

$$f(x,y) = y\sin x - \frac{1}{2}y^2 + C$$

So, $\vec{F}$ is conservative.

34. $\vec{F}(x,y,z) =< z^2 + 2xy,\ \ x^2 + 1,\ \ 2xz - 3 >$. Determine if it is possible to fine $f(x,y,z)$ such that $\nabla f = \vec{F}$.

$$f_x = z^2 + 2xy, \quad f(x,y,z) = \int (z^2 + 2xy)dx = z^2x + x^2y + C(y,z)$$

$$f_y = \frac{\partial}{\partial y}(z^2x + x^2y + C(y,z)) = x^2 + C_y(y,z) = x^2 + 1, \quad C_y(y,z) = 1$$

$$C(y,z) = \int dy = y + C(z), \quad f(x,y,z) = z^2x + x^2y + y + C(z)$$

$$f_z = 2zx + C'(z) = 2xz - 1,\ C'(z) = -1,\ C(z) = -z$$

$$f(x,y,z) = z^2x + x^2y + y - z$$

So, $\vec{F}$ does not have a potential function and it is not conservative.

38. $\vec{F}(x,y) =< \frac{1}{y},\ 2x >$

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{1}{y} \\ \dfrac{dy}{dt} = 2x \end{cases} \Rightarrow \frac{dx}{dy} = \frac{1}{y}\frac{1}{2x} \Rightarrow 2xdx = \frac{1}{y}dy \Rightarrow \int 2xdx = \int \frac{1}{y}dy \Rightarrow x^2 = \ln|y| + C$$

40. $\vec{F}(x,y) =< e^{-x},\ 2x >$

$$\begin{cases} \dfrac{dx}{dt} = e^{-x} \\ \dfrac{dy}{dt} = 2x \end{cases} \Rightarrow \frac{dx}{dy} = \frac{e^{-x}}{2x} = \frac{1}{2xe^x} \Rightarrow 2xe^x dx = dy \Rightarrow \int 2xe^x dx = \int dy$$

$$2xe^x - 2e^x = y + C, \quad y = 2xe^x - 2e^x + C$$