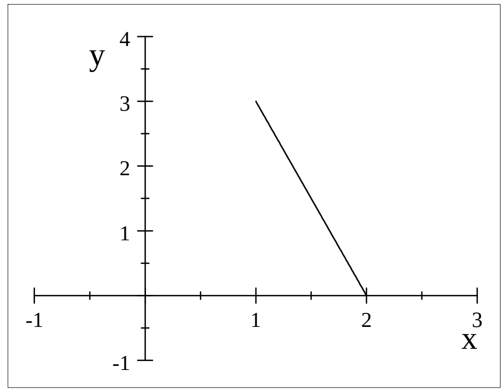
Homework Assignment 22 - (14.2) - Solutions

Page 1142 - turn in the problems with (*) - April 9
1-24: 6*, 8*, 12*, 14*, 18*, 22*
25-36: 28*, 35*, Extra points: 36

6.

$C : (2,0) \to (1,3),\ m = \frac{3-0}{1-2} = -3,\ \ y = -3(x-2)$

$\int_C 3y^2 dy = \int_0^3 3y^2 dy = 27$



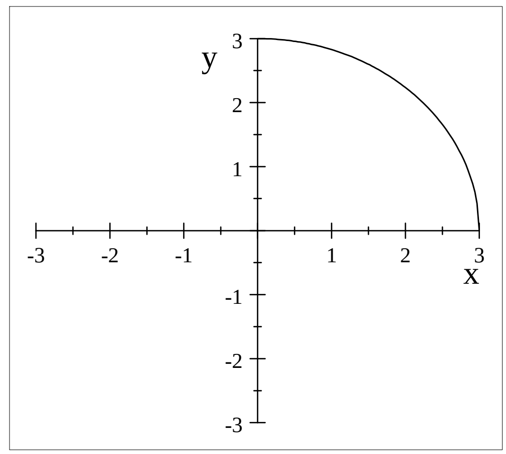
8.

$C : y = \sqrt{9-x^2}$ and x is from 0 to 3,

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1 + \left(\frac{x}{\sqrt{9-x^2}}\right)^2}\, dx$

$\int_C (3x - y) ds = \int_0^3 \left(3x - \sqrt{9-x^2}\right) \sqrt{1 + \left(\frac{x}{\sqrt{9-x^2}}\right)^2}\, dx$

$3\int_{\pi/2}^0 (9\cos(t) - 3\sin(t))dt = -18$



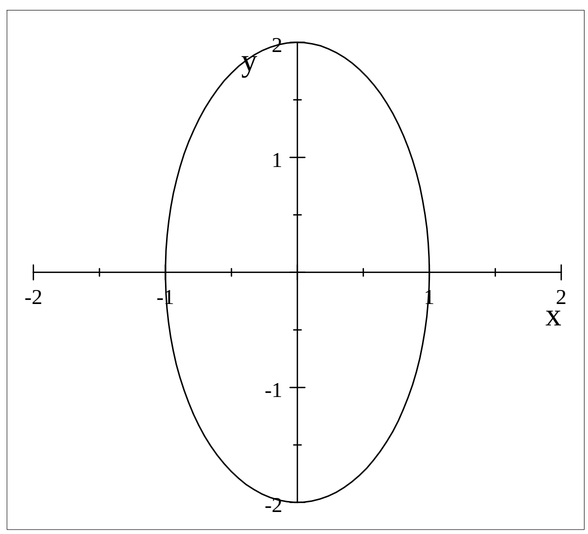
12.

$C : 4x^2 + y^2 = 4$ counter-clockwise

$r(t) = \langle \cos(t),\ 2\sin(t) \rangle,\ 0 \le t \le 2\pi$

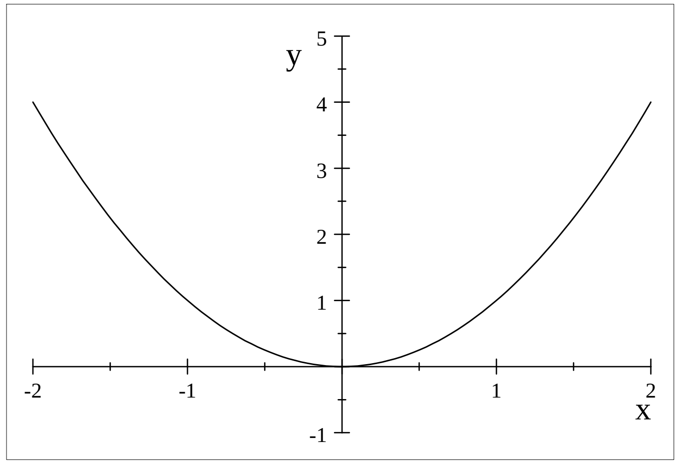
$r'(t) = \langle -\sin(t),\ 2\cos(t) \rangle$

$\int_0^{2\pi} \cos^2(t)(2\cos(t))dt = 0$



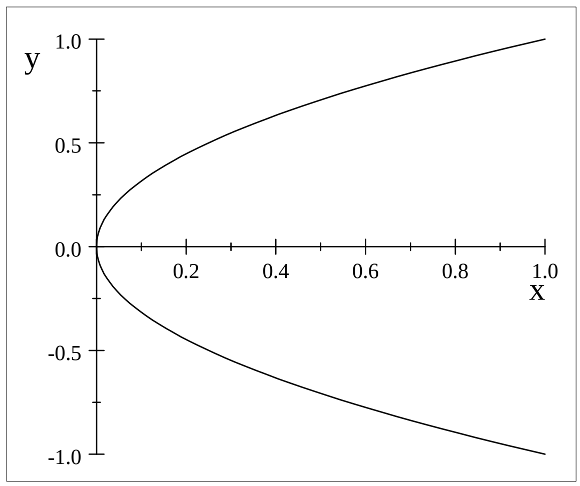
14.

$C : x : -2 \to 2,\ y = x^2,\ \dfrac{dy}{dx} = 2x$
$ds = \sqrt{1+4x^2}\,dx$
$\int_C 2x\,ds = \int_{-2}^{2} 2x\sqrt{1+4x^2}\,dx = 0$
(since $2x\sqrt{1+4x^2}$ is an odd function)



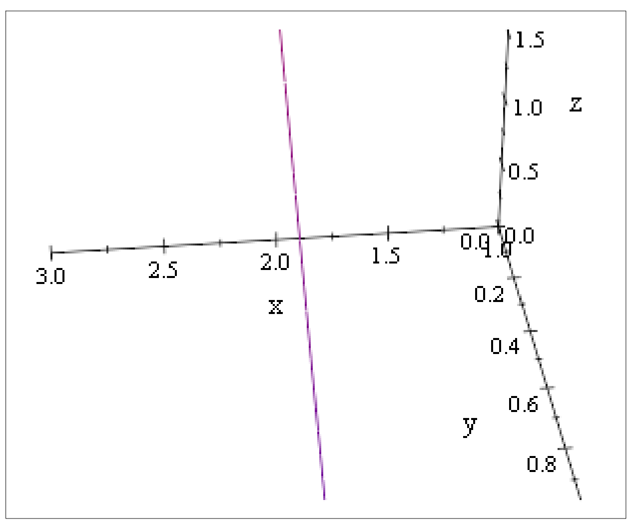
18. $C : x = y^2$, from $(1,1)$ to $(1,-1)$

$y : 1 \to -1,\ x = y^2$
$\int_C (x+y)\,dy = \int_1^{-1} (y^2+y)\,dy = -\frac{2}{3}$



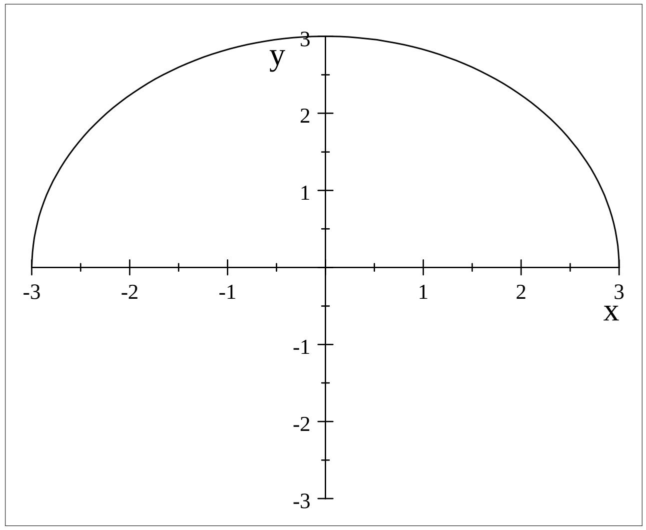
22. $C :$ a line segment, from $(2,1,0)$ to $(2,0,2)$

$r(t) = \langle 2, 1-t, 2t\rangle,\ 0 \le t \le 1$
$\int_0^1 4(2-2t)(2t)\,dt = \frac{8}{3}$



28. $\vec{F}(x,y) = \langle 2y, -2x\rangle,\ y = \sqrt{9-x^2}$, from $(-3,0)$ to $(3,0)$

$C:\ \vec{r}(t) = <3\cos t,\ 3\sin t>,\ t: \pi \to 0$
$\vec{r}\,'(t) = <-3\sin t,\ 3\cos t>$
$\int_C \vec{F} \cdot d\vec{r}$
$= \int_{\pi}^{0} \langle 2(3\sin t), -2(3\cos t)\rangle \cdot \langle -3\sin t, 3\cos t\rangle\, dt$
$= \int_{\pi}^{0} (-18\sin^2 t - 18\cos^2 t) dt$
$= -18\int_{\pi}^{0} dt = 18\pi$



Or, $\vec{r}(x) = \left\langle x, \sqrt{9-x^2} \right\rangle,\ x: -3 \to 3,\ \vec{r}\,'(x) = \left\langle 1, \frac{-2x}{2\sqrt{9-x^2}} \right\rangle = \left\langle 1, \frac{-x}{\sqrt{9-x^2}} \right\rangle$

$$\int_C \vec{F} \cdot d\vec{r} = \int_{-3}^{3} \left\langle 2\sqrt{9-x^2}, -2x \right\rangle \cdot \left\langle 1, \frac{-x}{\sqrt{9-x^2}} \right\rangle dx = \int_{-3}^{3} \left(2\sqrt{9-x^2} + \frac{-2x}{\sqrt{9-x^2}} \right)$$

$$= 2\int_{-3}^{3} \left(\frac{9}{\sqrt{9-x^2}} \right) = 18\pi$$

35. $\vec{F}(x,y,z) = \langle xy, 3z, 1\rangle,\ C:\ \vec{r}(t) = \left\langle \cos t,\ \sin t,\ 2t \right\rangle$, from $(1,0,0)$ to $(0,1,\pi)$

$0 \le t \le \pi,\ \vec{r}\,'(t) = \left\langle -\sin t,\ \cos t,\ 2 \right\rangle$
When $2t = 0,\ t = 0$, when $2t = \pi,\ t = \frac{\pi}{2}$.
$\int_C \vec{F} \cdot d\vec{r} = \int_0^{\pi/2} \langle \cos t \sin t, 3(2t), 1\rangle \cdot \langle -\sin t, \cos t, 2\rangle dt$
$= \int_0^{\pi/2} (-\cos t \sin^2 t + 6t\cos t + 2) dt$
$= -\frac{19}{3} + 4\pi$