Math 344 - Homework Assignment 2 - Due Jan. 23

1. MatLab programs for solving $Ax = b$ and $Ax = \begin{bmatrix} b_1 & \cdots & b_r \end{bmatrix}$ and finding A^{-1} by Gaussian Elimination and Backward Substitution:

a. The MatLab program **gaussbs.m** is designed to solve $Ax = b$ where A is an $n \times n$ matrix and b is an $n \times 1$ vector. It is an MatLab function and is called as:

[xv,umat,ct]=gaussbs(amat,b,n)

where matrix **amat** (A), vector **b** and dimension **n** are input and solution **xv**, upper triangular matrix **umat** and number **ct** of elementary row operation type I (exchange two rows) used are output. If A is singular, the program will be terminated.

b. The MatLab program **gaussbsmr.m** is designed to solve $AX = B = \begin{bmatrix} b_1 & \cdots & b_r \end{bmatrix}$ where A is an $n \times n$ matrix and B is an $n \times r$ matrix. It is an MatLab function and is called as:

xmat=gaussbsmr(amat,bmat,n,r)

where matrix **amat** (A), matrix **bmat** (B), dimension **n** and number **r** of vectors are input and solution matrix **xmat** is an output. If A is singular, the program will be terminated.

Down load a copy of each program from the course website and use them to solve the following problems.

a. Solve $Ax = b$ for Problems 1 and 4 on Page 158.

b. Solve A^{-1} for A in a.

c. Consider the matrix $A = [a_{ij}]$, where $a_{ij} = 1/(i + j - 1)$. Matrices of this type are called Hilbert matrix.

i. Give a 3×3 Hilbert matrix without using MatLab.

ii. In MatLab, a $n \times n$ Hilbert matrix can be formed by the command **hilb(n)**. Now we will use the program **gaussbs.m** to compute the determinant of a Hilbert matrix. Here is an example.

≫ **n=3**;
≫ **amat=hilb(n)**;
≫ **[xv,umat,ct]=gaussbs(amat,zeros(n,1),n)**; (**zeros(n,1)** is an $n \times 1$ zero vector)
≫ **ct** (check how many times rows have been exchanged)
≫ **ct=0** (so no exchange has been done)
≫ **diag(umat)** (check diagonal elements of U)
≫ **p=1**; **for i=1:n**, **p=p*umat(i,i)**; **end** (compute the determinant of U)
≫ **p** (get the determinant)
≫ **p=4.6296e-04** (get the determinant)

Now use above steps to compute the determinants of Hilbert matrices with sizes $n = 5$, $n = 10$, $n = 15$. What will you predict the determinant of a Hilbert matrix of size 16 or higher?

2. Page 168:
1(a) + (iv) Gassian elimination with complete pivoting.
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