

Math 344 - Homework Assignment 5 - Due Feb. 20

1. Page 234:

You may use MatLab programs Jacobi.m, Gauss_Seidel.m and SOR.m, posted on the course website: For each problem, turn in a copy of (1) MatLab commands used; and (2) results: k , $x^{(k)}$ and $\|x^{(k)} - x^{(k-1)}\|_\infty$.

8, 10, 11(8)

12(10) - you may try $\omega = 1, 1.05, 1.10, 1.15, 1.20, 1.25, 1.30, 1.35$. Choose more values if necessary. Find the interval (as small as possible) that contains ω_{opt} . Explain how you reach this result.

2. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -\frac{1}{2} & 1 & -\frac{1}{4} \\ 1 & -\frac{1}{2} & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 0.2 \\ -1.425 \\ 2 \end{bmatrix}$

- (1) Is A strictly diagonally dominant?
- (2) What is $\rho(T_{Jac})$? Would Jacobi Method converge for any $x^{(0)}$?
- (3) Use the program Jacobi.m to solve $Ax = b$ with (i) $tol = 10^{-2}$ and (ii) $tol = 10^{-3}$ and $kmax = 300$. Does Jacobi Method converge in both cases? If so, what are numbers of iterations?
- (4) What is $\rho(T_{GS})$? Would Gauss-Seidel Method converge for any $x^{(0)}$?
- (5) Use the program Gauss_Seidel.m to solve $Ax = b$ with (i) $tol = 10^{-2}$ and (ii) $tol = 10^{-3}$ and $kmax = 300$. Does Gauss_Seidel Method converge in both cases? If so, what are numbers of iterations?

3. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix}$. Apply the Gaussian Elimination without pivoting strategy to reduce A

to U .

What is the minimum number of multiplications needed to reduce A to U ? Show your work in detail.

Extra points: What the minimum number of multiplications needed to reduce A to U if

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1(n-1)} & a_{1n} \\ a_{21} & a_{22} & 0 & \cdots & 0 & 0 \\ a_{31} & 0 & a_{33} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{(n-1)1} & 0 & 0 & \cdots & a_{(n-1)(n-1)} & 0 \\ a_{n1} & 0 & 0 & \cdots & 0 & a_{nn} \end{bmatrix}$? Show your work in detail.