Review Notes for the Calculus I/Precalculus Placement Test - Fall, 2006

Part 6 -

1. **Quadratic functions: graph, zeros, vertex**

A **quadratic function** is a function of the form: $f(x) = ax^2 + bx + c,$ where $a,$ b and c are constants and $a \neq 0$. The **domain** of a quadratic function is the set of **all real numbers**. The graph of a quadratic function is a parabola which **opens up** (concave up) if $a > 0$ and **opens down** (concave down) if $a < 0$. Rewrite $f(x)$ in the form

$$f(x) = a(x-h)^2 + k, \text{ where } h \text{ and } k \text{ are constants,}$$

that can be done by the technique of **completing a square**:

$$f(x) = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}.$$

Then

$$h = -\frac{b}{2a} \text{ and } k = c - \frac{b^2}{4a},$$

the **vertex** of the parabola is $\left(h,\ k\right)$. The graph is **symmetric** about the line $x = h$. If the graph opens up, then k is the **minimum value** of f and if the graph is opens down, then k is the **maximum value** of f. Since a quadratic equation may have **a unique solution**, **two real solutions** or **no real solution**, the graph of a quadratic function may have a unique x –intercept, two x –intercepts or no x –intercept. If an x –intercept exists, it can be solved as follows:

$$a(x-h)^2 + k = 0, \quad (x-h)^2 = -\frac{k}{a}, \quad x - h = \pm\sqrt{-\frac{k}{a}}, \quad x = h \pm \sqrt{-\frac{k}{a}} \text{ if } -\frac{k}{a} \geq 0.$$

Example *Write each quadratic equation in the form of* $f(x) = a(x-h)^2 + k,$ *find its x –intercept(s) if possible and then sketch the graph.*

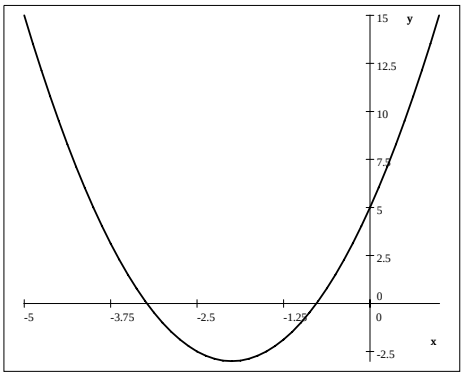
$$a.\ f(x) = 2x^2 + 8x + 5 \qquad b.\ f(x) = -3x^2 + 6x + 1$$

Solution *a.* $f(x) = 2x^2 + 8x + 5 = 2(x^2 + 4x) + 5 = 2(x^2 + 2(2x) + 4 - 4) + 5 = 2(x+2)^2 - 3.$ *The graph of* $f(x)$ *is a parabola which opens up and whose vertex is* $\left(-2,\ -3\right)$. *Since*

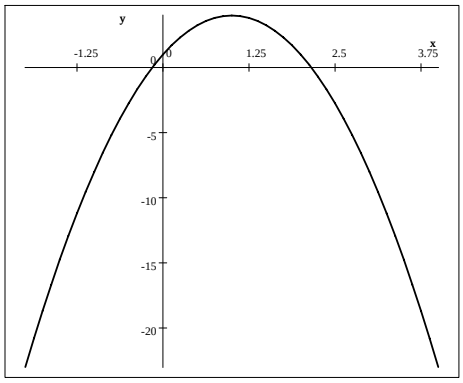
$$-\frac{k}{a} = -\frac{-3}{2} = \frac{3}{2} > 0,\ x - \text{intercepts are: } x_1 = -2 + \sqrt{\frac{3}{2}},\ \text{and } x_2 = -2 - \sqrt{\frac{3}{2}}.$$

b. $f(x) = -3x^2 + 6x + 1 = -3(x^2 - 2x) + 1 = -3(x^2 - 2(1)x + 1 - 1) + 1 = -3(x-1)^2 + 4.$ *graph of* $f(x)$ *is a parabola which opens down and whose vertex is* $\left(1,\ 4\right)$. *Since*

$$-\frac{k}{a} = -\frac{4}{-3} = \frac{4}{3} > 0, \quad x - \text{intercepts are: } x_1 = 1 + \sqrt{\frac{4}{3}} \text{ and } x_2 = 1 - \sqrt{\frac{4}{3}}.$$



a. $y = 2(x+2)^2 - 3$



b. $y = -3(x-1)^2 + 4$