Lecture 4: Nonlinear Filters

Math 490
Prof. Todd Wittman
The Citadel

Image Noise
- Physical effects (dust, raindrops)
- Electrical noise
  - Caused by fluctuations in electric circuit.
  - Thermal noise
  - Impulse noise
- Quantization noise
- Poor illumination

Applications
- Astronomy
- Microscopy
- Medical Imaging
- Computer Animation

Noise vs. Blur
- **Noise** adds static by randomly corrupting individual pixels.
- **Blur** smooths the edges by averaging over multiple pixels.

Mathematically speaking, noise is generally an additive process, while blur is generally multiplicative.

Artificial Noise
- We can add artificial noise to images using the Matlab `imnoise` command.
- We have to specify what type of noise we want and how much.
- The most common type of noise is **Gaussian** (white noise).

```matlab
A = imread('cameraman.tif');
B = imnoise(A, 'gaussian', 0, 0.3);
```

Additive Noise
- Gaussian Noise (**MRI**)
  \[ f = g + N(0, \sigma) \]
- Poisson Noise (**PET**)
  \[ f = \text{Poisson}(g) \]

```matlab
A = imread('cameraman.tif');
B = imnoise(A, 'gaussian', 0, 0.3);
C = imnoise(A, 'poisson');
```
Non-additive Noise

- Salt & Pepper Noise (Reset random pixels)
- Speckle Noise (Ultrasound) \( f = g(1 + U(0, \sigma)) \)

```
A = imread('cameraman.tif'); B = imnoise(uint8(A),'salt & pepper',0.1);
imagesc(B);
```

Filters

- A filter is a process that removes or enhances some feature of an image.
- Commonly, the word "filter" describes an operation on the neighborhood of an image.

```
Median Filter

The median filter computes the median of the neighborhood centered over the pixel.
Compared to the mean filter, the median filter preserves edges better.

\[
\begin{array}{ccc}
255 & 255 & 0 \\
255 & 100 & 0 \\
255 & 255 & 0 \\
\end{array}
\]
Mean = \( \frac{255+255+0+255+100+0+255+255+0}{9} \) = 152.8
Median = 0, 0, 0, 100, 255, 255, 255, 255 = 255

Visualizing Noise

- A common way to test denoising algorithms is to subtract the denoised image from the noisy image.
- This subtracted image is called the residual.
- If the algorithm did a good job denoising, then we should only see random dots and no structure.

```
Median Filter

The median filter and its variants are particularly well-suited for salt & pepper noise.
The Matlab command `nlfilter` calculates nonlinear functions on neighborhoods. It is much slower than `imfilter`.
B = nlfilter(A, [3,3], @(x) median(x(:)));
Can also use the command `medfilt2`.
```

Linear vs. Nonlinear

- A linear filter can be expressed as a convolution (weighted average over a neighborhood):
  \( g = f * w \)
- Any filter that cannot be written as a convolution is called a nonlinear filter.
- Linear filters can be computed very quickly using Fourier Transforms by The Convolution Theorem.
- Nonlinear filters will generally be much slower.
Quantifying Noise
- There are two major statistics used to determine the amount of noise in an image.
- Both statistics compare the noisy image $f$ to an ideal noise-free image $f_{ideal}$.
- Both statistics are easier to write using the Frobenius norm:
  \[ \|A\|_F = \sqrt{\sum_{i,j} a_{ij}^2} \]

1. Root Mean Square Error (RMSE)
   \[ RMSE = \frac{1}{N} \| f - f_{ideal} \|_F \quad \text{where } N = \#\text{pixels in image} \]

2. Signal-to-Noise Ratio (SNR)
   \[ SNR = 20 \log \left( \frac{\| f_{ideal} \|_F}{\| f - f_{ideal} \|_F} \right) \]
   Larger SNR = Less Noise

Thought Exercise
- A noisy brain MRI is segmented into 4 classes using K-means clustering.
  \[ K = \text{kmeans}(A(:), 4); \]
  \[ B = \text{reshape}(K, \text{size}(A)); \]
- The image $B$ takes on integer values 1-4.

- Mean and median filters are not appropriate for this type of data. Can you explain why?

Thought Exercise
- The data is qualitative. The values 1,2,3,4 are just labels and the order is irrelevant.
- We could run a __________ filter on the data.
  \[ C = \text{nlfilter}(B, [3,3], @(x)\________)(x(:)); \]

Beyond Local
- Suppose Lena's left eye was obscured by noise.
- A neighborhood around the eye will not tell us the eye's true color.
- Solution: Look at the other eye!

Image Similarity
- Natural images tend to have similar neighborhoods.

Nonlocal-Means
- (Buades-Coll-Morel, 2005) proposed a denoising method that averages over all pixels in the image that have a similar neighborhood.

- They call their method Nonlocal (NL) Means.
  \[ NL(x) = \frac{\sum w(x,q) f(q)}{\sum w(x,q)} \]
  where $w$ is a weight that judges the similarity between the neighborhoods.
  Similar $\rightarrow w=1$, Different $\rightarrow w=0$
Nonlocal-Means

- Let $N(x)$ denote the neighborhood around pixel $x$.
- We want $w(x,q)=1$ if the neighborhoods $N(x)$ and $N(q)$ are similar.
- And we want $w(x,q)=0$ if the neighborhoods $N(x)$ and $N(q)$ are completely different.

$$w(x, q) = \exp\left(-\frac{||N(x) - N(q)||^2}{h^2}\right)$$

- The parameter $h$ controls the similarity.
- Generally choose square neighborhood, e.g. 5x5.
- Can add Gaussian weights to norm to emphasize center.

Weights from Clean Images

Weights from Noisy Images

NL-Means

- Smaller neighborhoods remove noise better, but may over-smooth the image.

NL-Means

- Many consider NL-Means and its variants to be the best denoising method for natural images.

NL-Means

- NL-Means performs best on images with repeating patterns.
Implementation

NL-Means is painfully slow. There are 2 tricks to speed it up:

1. **Search Somewhat Local** -- Restrict the neighborhood comparisons to a window around the current pixel, rather than the whole image.

2. **Save Pixel Comparisons** -- Store all pixel-to-pixel comparisons in an efficient data structure to avoid making unnecessary passes through the image.

Color Images

- We can extend NL-Means to color images by comparing 3D neighborhood cubes.
- NL-Means actually achieves better results on color images than on grayscale. We can shrink the neighborhood size because of the extra information.

3D NL-Means

- NL-Means extends to 3D for surface denoising.
- NL-Means recovers corners and flattens smooth regions better than many denoising methods.

Ultrasound

- Nonlocal methods are good at removing the speckle noise in Ultrasound images.
- NL-Means is particularly good for microscopy.

More Neighborhoods

- Standard NL-Means only compares neighborhoods under translation.
- We could also consider neighborhoods under scaling and rotation transformations.
- But this will slow down the computation.

Use with Caution

- Nonlocal methods are powerful denoisers, but they have to be used carefully.
  - Exploiting image similarity could erase small-scale features that only occur in one part of the image.
  - But these small features are often the most important part in some applications. For example, in medical imaging we look for tumors, hemorrhages, muscle tears, bone fractures, etc.