

Solutions to Practice Problem Set 3 for the Derivative Gateway Test - 2003

Find the derivatives of the following functions:

1. $y = \frac{t^2 + 2t + e}{\pi} = \frac{1}{\pi}(t^2 + 2t + e)$

$$\frac{dy}{dt} = \frac{1}{\pi}(2t + 2)$$

2. $f(x) = \ln \left| \frac{2x+5}{7x+9} \right| = \ln|2x+5| - \ln|7x+9|$

$$f'(x) = \frac{2}{2x+5} - \frac{7}{7x+9}$$

3. $z = we^{-\frac{1}{w}} = we^{-w^{-1}}$

$$\frac{dz}{dw} = e^{-1/w} + we^{-1/w}(-(-1)w^{-2}) = e^{-1/w}(1 + \frac{1}{w})$$

4. $v = 2 \sin u \cos u = \sin(2u)$

$$\frac{dv}{du} = \cos(2u)(2) = 2 \cos(2u)$$

5. $r = \cos^2 \theta - \sin^2 \theta = \cos(2\theta)$

$$\frac{dr}{d\theta} = -\sin(2\theta)(2) = -2 \sin(2\theta)$$

6. $y = 2 \cos^5 \left(\sqrt{\frac{1}{t}} \right) = 2[\cos(t^{-1/2})]^5$

$$\frac{dy}{dt} = 2(5) \cos^4 \left(\sqrt{\frac{1}{t}} \right) \left(-\sin \left(\sqrt{\frac{1}{t}} \right) \right) \left(-\frac{1}{2} t^{-3/2} \right) \text{ or } 5 \sqrt{\frac{1}{t^3}} \cos^4 \left(\sqrt{\frac{1}{t}} \right) \sin \left(\sqrt{\frac{1}{t}} \right)$$

7. $H = 2^x \csc x$

$$\frac{dH}{dx} = 2^x \ln 2 \csc x + 2^x (-\cot x \csc x) \text{ or } 2^x \csc x (\ln 2 - \cot x)$$

8. $g(\theta) = \frac{\tan \theta}{\theta^2 + 9}$

$$g'(\theta) = \frac{\sec^2 \theta (\theta^2 + 9) - \tan \theta (2\theta)}{(\theta^2 + 9)^2} \text{ or } \frac{(\theta^2 + 9) \sec^2 \theta - (2\theta) \tan \theta}{(\theta^2 + 9)^2}$$

9. $y = e^{\sec \theta}$

$$\frac{dy}{d\theta} = e^{\sec\theta}(\sec\theta \tan\theta)$$

10. $y = \sec e^\theta$

$$\frac{dy}{d\theta} = \sec(e^\theta)(\tan(e^\theta))(e^\theta) \stackrel{\text{or}}{=} e^\theta \sec(e^\theta) \tan(e^\theta)$$