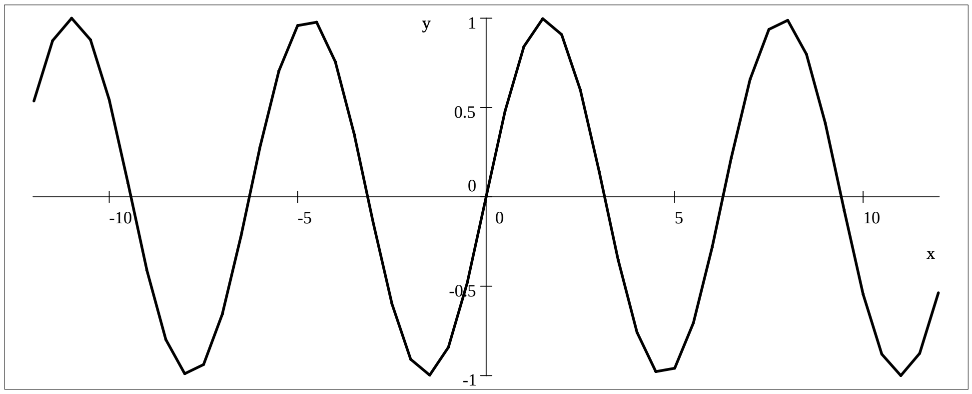
Review Notes for the Calculus I/Precalculus Placement Test - Fall, 2006

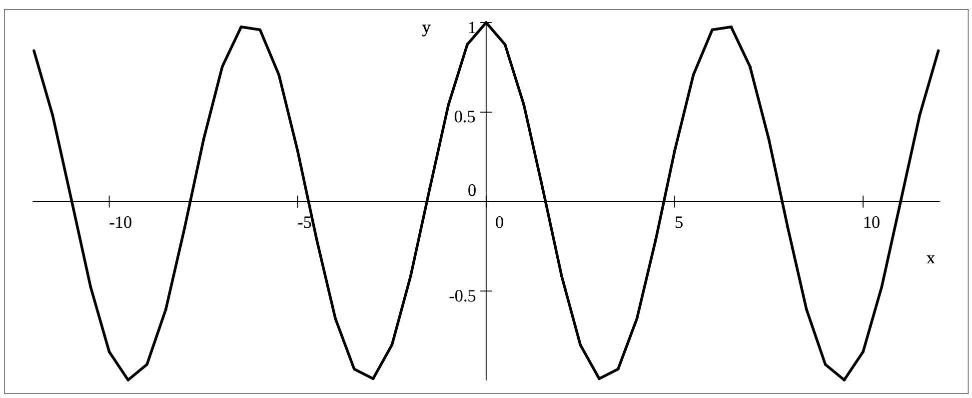
Part 10 -

1. **Trigonometric functions:** $\sin x$, $\cos x$, $\tan x$, **domain, period and graph**
   a. $f(x) = \sin x$.
   The **domain** of $\sin x$ is the set of **all real numbers** and the **period** T is 2π. Values of $\sin x$ are in $[-1,\ 1]$. The **graph** of $\sin x$ is given below.



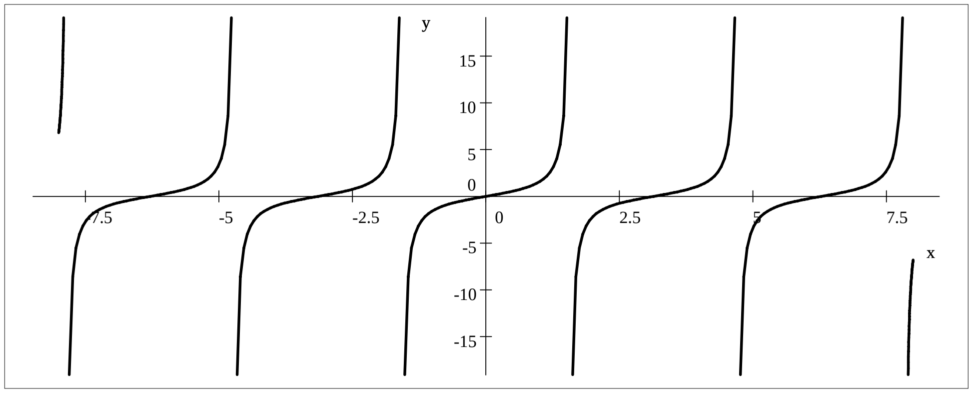
$y = \sin x$

   b. $f(x) = \cos x$.
   The **domain** of $\cos x$ is the set of **all real numbers** and the **period** T is 2π. Values of $\cos x$ are in $[-1,\ 1]$. The **graph** of $\cos x$ is given below.



$y = \cos x$

   c. $f(x) = \tan x = \dfrac{\sin x}{\cos x}$
   The **domain** of $\tan x$ is the set of all real numbers **except all zeros of** $\cos$**x**. The **period** T is π.
   The **graph** of $\tan x$ is given below.



$y = \tan x$

Example *Evaluate the following.*

i. $\sin(390^\circ) = \sin(360^\circ + 30^\circ) = \sin(30^\circ) = \dfrac{1}{2}$

ii. $\cos\dfrac{11\pi}{4} = \cos\left(2\pi + \dfrac{3\pi}{4}\right) = \cos\left(\dfrac{3\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

iii. $\tan\dfrac{4\pi}{3} = \tan\left(\pi + \dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{3}\right) = \sqrt{3}$

Example *Give the domain of* $\tan x$ *for* $-4\pi \le x \le 4\pi$.

Solution *The domain of* $\tan x$ *is the set of all real numbers in* $[-4\pi,\ 4\pi]$ *except zeros of* $\cos x$ *in this interval. Zeros of* $\cos x$ *in* $[-4\pi,\ 4\pi]$ *are:*

$$\frac{\pi}{2},\quad \frac{3\pi}{2},\quad 2\pi + \frac{\pi}{2} = \frac{5\pi}{2},\quad 2\pi + \frac{3\pi}{2} = \frac{7\pi}{2},\quad -2\pi + \frac{\pi}{2} = -\frac{3\pi}{2},$$
$$-2\pi + \frac{3\pi}{2} = -\frac{\pi}{2},\quad -4\pi + \frac{3\pi}{2} = -\frac{5}{2}\pi,\quad -4\pi + \frac{\pi}{2} = -\frac{7}{2}\pi$$

The domain of $\tan x$ *for* $-4\pi \le x \le 4\pi$ *is the set of all real numbers in* $[-4\pi,\ 4\pi]$ *except values*

$$\frac{\pi}{2},\quad \frac{3\pi}{2},\quad \frac{5\pi}{2},\quad \frac{7\pi}{2},\quad -\frac{3\pi}{2},\quad -\frac{\pi}{2},\quad -\frac{5}{2}\pi,\quad -\frac{7}{2}\pi.$$

2. **Amplitudes, periods, graphs of** $A\sin\omega x,\ A\cos\omega x$

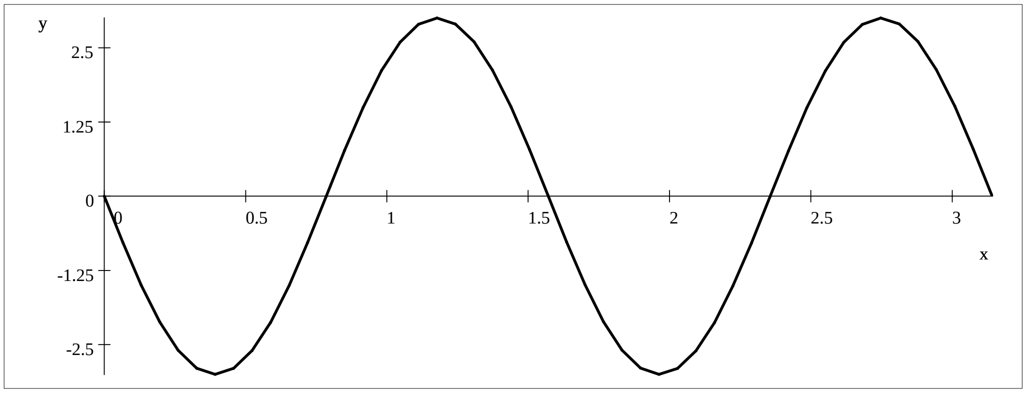
Let ω be positive. Then the **amplitude** and **period** of $y = A\sin\omega x$ and $y = A\cos\omega x$ are given by

$$\textbf{Amplitude} = |A|, \quad \text{and} \quad \textbf{period} = T = \frac{2\pi}{\omega}.$$

Example *Determine the amplitude and period of each function and then sketch its graph for two periods.*

a. $f(x) = -3\sin(4x)$

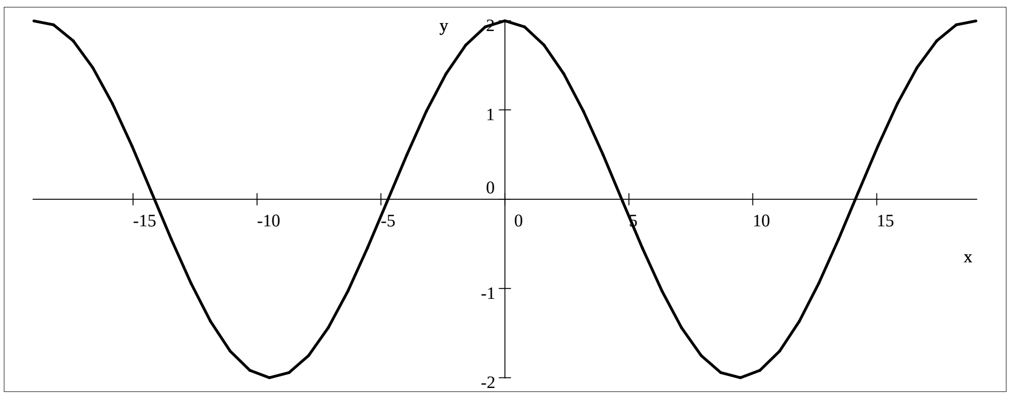
$f(x) = -3\sin(4x)$, *amplitude is* $|A| = |-3| = 3$, *period is* $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$. *Choose two periods:* $[0,\ \pi]$.



$$y = -3\sin(4x)$$

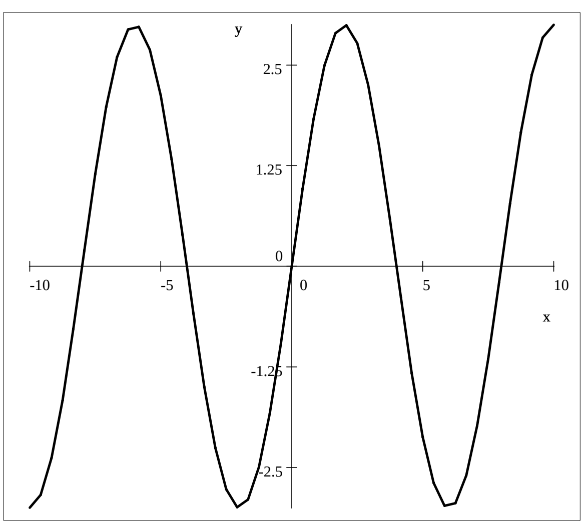
b. $f(x) = 2\cos\left(\frac{1}{3}x\right)$

$f(x) = 2\cos\left(\frac{1}{3}x\right)$, *amplitude is* $|A| = |2| = 2$, *period is* $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{1}{3}} = 6\pi$. *Choose two periods:* $\left[-6\pi,\ 6\pi\right]$.

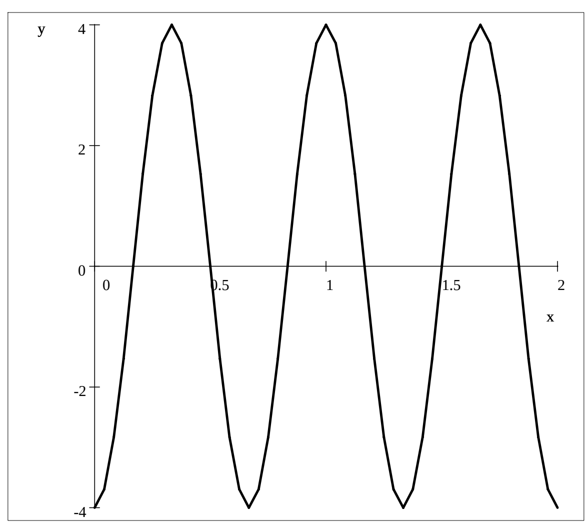


$$y = 2\cos\left(\tfrac{1}{3}x\right)$$

Example *Graphs of* $f(x) = A\sin\omega x$ *or* $f(x) = A\cos\omega x$ *are given below. Find a function for each graph. Estimate A and ω as close as possible.*



a.



b.

Solution *Find first the amplitude and period of the function.*

a. *Since* $(0,0)$ *is on the graph, it is the graph of* $f(x) = A\sin(\omega x)$. $A = 3$. *Since* $T = 8 = \frac{2\pi}{\omega}$, $\omega = \frac{2\pi}{8} = \frac{\pi}{4}$. *So,*

$$f(x) = 3\sin\left(\frac{\pi}{4}x\right).$$

b. *Since* $(0,-4)$ *is on the graph, it is the graph of* $f(x) = A\cos(\omega x)$. *Since* $-A = 4$, $A = -4$. *Since*

$$T \approx 0.65 = \frac{2\pi}{\omega}, \quad \omega \approx \frac{2\pi}{0.65} = 3.076923\pi.$$

So,

$$f(x) = -4\cos(3.076923\pi x)$$

3. **Identities:** $\sin(2\theta)$**,** $\cos(2\theta)$
Sum and Difference Formulas:

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Now let $\alpha = \beta = \theta$. The following **double-angle formulas** can be derived directly.
Double-angle formulas:

$\sin(2\theta) = 2\sin\theta\cos\theta$
$\cos(2\theta) = \cos^2\theta - \sin^2$

By the identity $\sin^2\theta + \cos^2\theta = 1$, $\cos(2\theta)$ can be also written as

$$\cos(2\theta) = 1 - 2\sin^2\theta \quad \text{and} \quad \cos(2\theta) = 2\cos^2\theta - 1.$$

Example *Let* θ *be acute* $\left(0 < \theta < \frac{\pi}{2}\right)$. *If* $\sin\theta = \frac{3}{4}$, *find* $\sin(2\theta)$ *and* $\cos(2\theta)$.

Solution *First find* $\cos\theta$. $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{3}{4}\right)^2} = \frac{\sqrt{7}}{4}$. *Then*

$$\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(\frac{3}{4}\right)\left(\frac{\sqrt{7}}{4}\right) = \frac{3\sqrt{7}}{8}$$

$$\cos(2\theta) = 1 - 2\sin^2\theta = 1 - 2\left(\frac{3}{4}\right)^2 = -\frac{1}{8}$$