Review Notes for the Calculus I/Precalculus Placement Test - Fall, 2006

Part 2 -

1. **Integer exponents**
   a. **Integer exponents:** $a^n = (a)(a)\cdots(a)$, a product of n factors of a.
   a is called a base and n is called an exponent or power.
   Rules:

(1)	$a^{-n} = \frac{1}{a^n}$	(5)	$(ab)^n = a^n b^n$
(2)	$a^0 = 1$ if $a \neq 0$	(6)	$\frac{a^m}{a^n} = a^{m-n}$
(3)	$a^m a^n = a^{m+n}$	(7)	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
(4)	$(a^m)^n = a^{mn}$	(8)	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Example *Evaluate each expression.*

i. 2^3

$$2^3 = 8$$

ii. $(-2)^3$

$$(-2)^3 = (-1)^3 2^3 = -8$$

iii. -2^4

$$-2^4 = -16$$

iv. $(-2a)^3$

$$(-2a)^3 = (-1)^3 2^3 a^3 = -8a^3$$

v. $\frac{1}{2^3}$

$$\frac{1}{2^3} = \frac{1}{8}$$

vi. $\left(\frac{2}{3}\right)^3$

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

vii. $\left(\frac{2}{3}\right)^{-3}$

$$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$$

Example *Simplify each expression. Express the answer with only positive exponents.*

i. $\frac{2^6}{2^3}$

$$\frac{2^6}{2^3} = 2^{6-3} = 2^3 = 8$$

ii. $\frac{x^{-2}}{x^{-5}}$

$$\frac{x^{-2}}{x^{-5}} = x^{-2-(-5)} = x^3$$

iii. $\dfrac{a^5b^{-2}}{(a^3b)^3}$

$$\frac{a^5b^{-2}}{(a^3b)^3} = \frac{a^5b^{-2}}{a^9b^3} = a^{5-9}b^{-2-3} = a^{-4}b^{-5} = \frac{1}{a^4b^5}$$

iv. $\left(\dfrac{2x^{-3}y}{3xy^{-2}}\right)^{-2}$

$$\left(\frac{2x^{-3}y}{3xy^{-2}}\right)^{-2} = \left(\frac{3xy^{-2}}{2x^{-3}y}\right)^{2} = \frac{3^2x^2y^{-4}}{2^2x^{-6}y^2} = \frac{9}{4}x^{2-(-6)}y^{-4-2} = \frac{9}{4}x^8y^{-6} = \frac{9x^8}{4y^6}$$

b. **Radicals and rational exponents:**
nth Roots: the principal *n*th root of a real number *a* where $n \geq 2$ an integer, symbolized by $\sqrt[n]{a}$, is identified as $\sqrt[n]{a} = b$ means $a = b^n$.
Rules:

(1)	$\sqrt[n]{a^n} = \begin{cases} a, & \text{if } n \geq 3 \text{ is odd} \\ \lvert a \rvert, & \text{if } n \geq 2 \text{ is even} \end{cases}$	(3)	$\sqrt[n]{\dfrac{a}{b}} = \left(\dfrac{a}{b}\right)^{1/n} = \dfrac{a^{1/n}}{b^{1/n}}$
(2)	$\sqrt[n]{ab} = (ab)^{1/n} = a^{1/n}b^{1/n}$	(4)	$\sqrt[n]{a^m} = a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$

Example *Simplify radicals and express each answer with rational exponents.*

i. $\sqrt[4]{16} = 2$

$$\sqrt[4]{16} = (2^4)^{1/4} = 2$$

ii. $\sqrt[3]{-16a^4}$

$$\sqrt[3]{-16a^4} = (-1)^{1/3}(2^4)^{1/3}(a^4)^{1/3} = -2^{4/3}a^{4/3}$$

iii. $\sqrt[3]{\dfrac{8x^5}{27y^6}}$

$$\sqrt[3]{\frac{8x^5}{27y^6}} = \left(\frac{2^3x^5}{3^3y^6}\right)^{1/3} = \frac{2x^{5/3}}{3xy^{6/3}} = \frac{2}{3}x^{5/3-1}y^{-2} = \frac{2}{3}\frac{x^{2/3}}{y^2}$$

iv. $\dfrac{\sqrt{3xy^3}\,\sqrt{2x^2y}}{\sqrt{6x^3y}}$

$$\frac{\sqrt{3xy^3}\,\sqrt{2x^2y}}{\sqrt{6x^3y}} = \frac{(3xy^3)^{1/2}(2x^2y)^{1/2}}{(6x^3y)^{1/2}} = \frac{3^{1/2}x^{1/2}y^{3/2}(2^{1/2}x^{2/2}y^{1/2})}{6^{1/2}x^{3/2}y^{1/2}} = x^{1/2+1-3/2}y^{3/2+1/2-1/2}$$

$$= x^0y^{3/2} = y^{3/2}$$

v. $2x\sqrt{x^2+1} + \dfrac{1}{3}x^2(x^2+1)^{-1/2}(2x)$

$$2x(x^2+1)^{1/2} + \frac{1}{3}x^2(x^2+1)^{-1/2}(2x) = 2x\left(\sqrt{x^2+1} + \frac{1}{3}\frac{x^2}{\sqrt{x^2+1}}\right)$$

$$= 2x\left(\sqrt{x^2+1}\,\frac{3\sqrt{x^2+1}}{3\sqrt{x^2+1}} + \frac{1}{3}x^2\frac{1}{\sqrt{x^2+1}}\right) = 2x\left(\frac{3(x^2+1)+x^2}{3\sqrt{x^2+1}}\right) = \frac{2x(4x^2+3)}{3(x^2+1)^{1/2}}$$

2. **Polynomial algebra:** $+, -, \times$

An n**th degree polynomial in** x : $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ where $a_0, a_1, \ldots, a_n$ are constants, $a_n \neq 0$, and $n \geq 0$ is an integer. Constants $a_0, a_1, \ldots, a_n$ are called coefficients of the polynomial and n is called the degree of the polynomial.

Polynomial algebra:

Polynomials are added and subtracted by combining like terms. Polynomials are multiplied using the rules of exponents and distributive properties.

Example *Let* $P = 8x^3 - 2x^2 + 6x - 2$ *and* $R = 3x^4 - 2x^3 + x^2 + x$. *Compute the following.*

a. $P + R$

$$\begin{aligned} P + R &= (8x^3 - 2x^2 + 6x - 2) + (3x^4 - 2x^3 + x^2 + x) \\ &= 3x^4 + (8 - 2)x^3 + (-2 + 1)x^2 + (6 + 1)x - 2 \\ &= 3x^4 + 6x^3 - x^2 + 7x - 2 \end{aligned}$$

b. $P - R$

$$\begin{aligned} P - R &= (8x^3 - 2x^2 + 6x - 2) - (3x^4 - 2x^3 + x^2 + x) \\ &= -3x^4 + (8 + 2)x^3 + (-2 - 1)x^2 + (6 - 1)x - 2 \\ &= -3x^4 + 10x^3 - 3x^2 + 5x - 2 \end{aligned}$$

c. PR

$$\begin{aligned} PR &= (8x^3 - 2x^2 + 6x - 2)(3x^4 - 2x^3 + x^2 + x) \\ &= 8x^3(3x^4 - 2x^3 + x^2 + x) - 2x^2(3x^4 - 2x^3 + x^2 + x) \\ &\quad + 6x(3x^4 - 2x^3 + x^2 + x) - 2(3x^4 - 2x^3 + x^2 + x) \\ &= 24x^7 - 16x^6 + 8x^5 + 8x^4 - (6x^6 - 4x^5 + 2x^4 + 2x^3) + 18x^5 \\ &\quad - 12x^4 + 6x^3 + 6x^2 - (6x^4 - 4x^3 + 2x^2 + 2x) \\ &= 24x^7 + (-16 - 6)x^6 + (8 + 4 + 18)x^5 + (8 - 2 - 12 - 6)x^4 \\ &\quad + (-2 + 6 + 4)x^3 + (6 - 2)x^2 - 2x \\ &= 24x^7 - 22x^6 + 30x^5 - 12x^4 + 8x^3 + 4x^2 - 2x \end{aligned}$$