Review Notes for the Calculus I/Precalculus Placement Test - Fall, 2006

Part 5 -

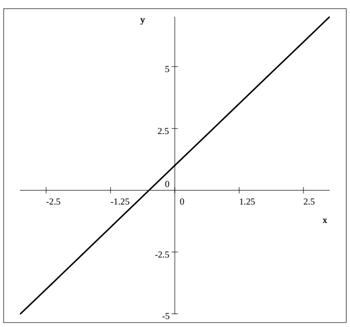
1. **Linear functions: graph, slope,** x, y **–intercepts**
   Linear Functions: $f(x) = mx + b$, where m and b are real numbers.
   The **domain** of a linear function is the set of **all real numbers**. The **graph** of a linear function is **a non-vertical line** with **slope** m and **y –intercept** b. A linear function is **increasing** if $\mathbf{m > 0}$, **decreasing** if $\mathbf{m < 0}$, and a **constant** if $\mathbf{m = 0}$.

Example *Sketch the graph of the function f.*

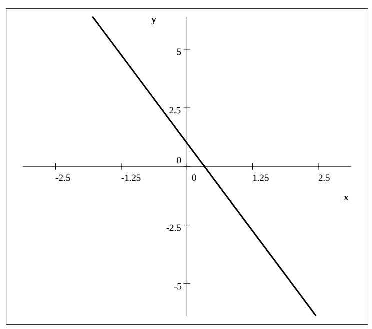
a. $f(x) = 2x + 1$ *b.* $f(x) = -3x + 1$ *c.* $f(x) = 1$

Solution *Note that two points determine a line. So, find two points on the line and then sketch the line passing through these two points. Remember that the y-intercept* $(0,\ b)$ *is also a point on the line.*

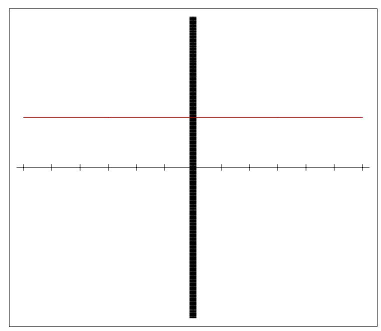
a. Two points: $(0,\ 1)$ and $(1,\ 3)$
b. Two points: $(0,\ 1)$ and $(1,\ -2)$
c. Because $m = 0$, the graph of $y = 1$ is the horizontal line passing through $(0,\ 1)$.



a. $f(x) = 2x + 1$



b. $f(x) = -3x + 1$



c. $f(x) = 1$

A **vertical** line is the graph of a **linear equation** $x = a$. A vertical line cannot be the graph of a linear function. Recall that a function assigns each x exactly one $y = f(x)$ value. So, points $(a,\ y_1)$ and $(a,\ y_2)$ where $y_1 \neq y_2$ should not be on the graph of a function. Points on a vertical line $x = a$ are $(a, 1),\ (a, 2),\ \cdots$. There are **infinitely many** y values which are assigned to $x = a$.

linear equation	**linear function**
$y = mx + b$	$f(x) = mx + b$
its graph is a line	the graph of f is a non-vertical line

a. **Slope of a line**
   Let $P = (x_1,\ y_1)$ and $Q = (x_2,\ y_2)$ be two distinct points $(x_1 \neq x_2)$. The **slope** m of the line L containing points P and Q is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

   If $x_1 = x_2$, then m is not defined. So, the slope of a vertical line is not defined. Now let $x_1 < x_2$. Then
   $m > 0$ if and only if $y_2 > y_1$ (the corresponding linear function is **increasing**);

$m < 0$ if and only if $y_2 < y_1$ (the corresponding linear function is **decreasing**); and
$m = 0$ if and only if $y_2 = y_1$ (the corresponding linear function is **a constant**).

b. **Intercepts of a line** $y = mx + b$
The $x-$ intercept: $(a,\ 0)$ where $a = -\dfrac{b}{m}$ if $m \neq 0$ (or the line is not horizontal)
The $y-$ intercept: $(0,\ b)$

2. **Equation of a line: slope-intercept form, point-slope form**
   a. The equation of a **horizontal line** with the y –intercept b : $y = b$.
   b. The equation of a **vertical line** with the x –intercept a : $x = a$.
   c. The **slope-intercept form** of a line with slope m and y-intercept $(0,\ b)$: $y = mx + b$
   d. The **point-slope form** of a line with slope m and passing through the point $(x_1,\ y_1)$:

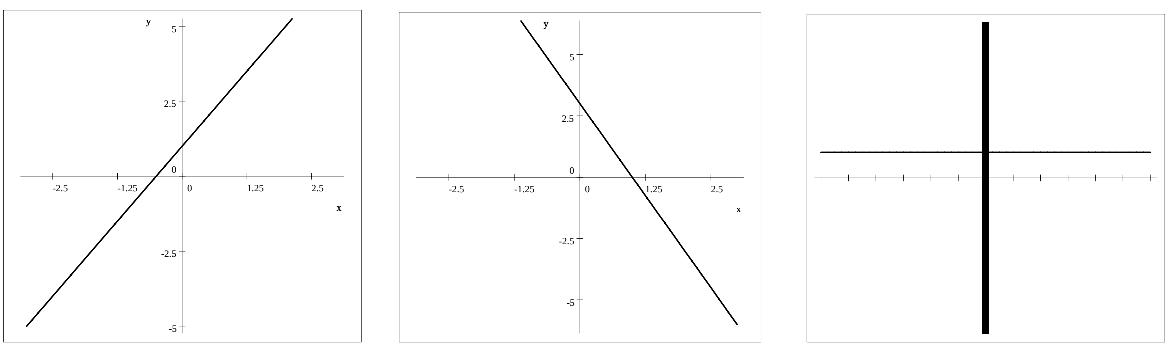
$$y - y_1 = m(x - x_1)$$

   e. The equation of a line **containing two points**: $(x_1,\ y_1)$ and $(x_2,\ y_2)$ where $x_1 \neq x_2$:

$$y - y_1 = m(x - x_1) \text{ where } m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Let lines L_1 and L_2 be $y = m_1 x + b_1$ and $y = m_2 + b$, respectively. Observe that $m_1 = m_2$ if and only if lines L_1 and L_2 are **parallel**; and $m_1 = -\dfrac{1}{m_2}$ or $m_1 m_2 = -1$ if and only if two lines which are not parallel to the axes, are **perpendicular.**

Example *Find the equation of each given line.*



a. b. c.

a. Pick (**as accurate as possible**) two points on the line: $(0,\ 1)$, $\left(-\frac{1}{2},\ 0\right)$. Compute the slope:

$$m = \frac{0-1}{-\frac{1}{2}-0} = 2, \qquad \text{the equation of the line in the **slope-intercept** form: } y = 2x + 1$$

b. Pick (**as accurate as possible**) two points on the line: $(0,\ 3)$, $(1,\ 0)$.

$$m = \frac{0-3}{1-0} = -3, \qquad \text{the equation of the line in the **slope-intercept** form: } y = -3x + 3$$

c. The y –intercept of the line is $(0,\ 1)$ and the slope $m = 0$, the equation of the line: $y = 1$.

Example *Find the slope and y –intercept of each line.*

$$a.\ \frac{1}{2}y = x - 1 \qquad b.\ x + 2y = 4 \qquad c.\ y = -1 \qquad d.\ 3x + 2y + 4 = 0$$

Solution *Express the equation in the form of $y = mx + b$. Then m is the **slope** and b is the y –**intercept**.*

a. $\frac{1}{2}y = x - 1,\ y = 2(x-1) = 2x - 2.$ The slope is 2 and the y –intercept is -2.

b. $x + 2y = 4,\ 2y = -x + 4,\ y = -\frac{1}{2}x + \frac{1}{2}(4) = -\frac{1}{2}x + 2.$ The slope is $-\frac{1}{2}$ and y –intercept is 2.

c. $y = -1 = 0x - 1.$ The slope is 0 and y –intercept is -1.

d. $3x + 2y + 4 = 0,\ 2y = -3x - 4,\ y = -\frac{3}{2}x - 2.$ The slope is $-\frac{3}{2}$ and y –intercept is -2.

Example *Find the equation of the line L which passes through the point $(-1,\ 2)$ and is parallel to the line $2x + 3y = 1$.*

Solution *Find the slope of the given line:* $3y = -2x + 1,\ y = -\frac{2}{3}x + \frac{1}{3}.$ *The slope is* $-\frac{2}{3}.$ *Use the slope-point form to express the equation of L :*

$$y - 2 = -\frac{2}{3}(x+1), \quad or \quad y = -\frac{2}{3}x - \frac{2}{3} + 2 = -\frac{2}{3}x + \frac{4}{3}.$$

Example *Find the equation of the line L which is perpendicular to the line $y = 2x + 1$ and whose x –intercept is -2.*

Solution *The line L passes through the point $(-2,\ 0)$ and has the slope $-\left(\frac{1}{2}\right)$. So, the equation of the line L is:*

$$y - 0 = -\frac{1}{2}(x+2), \qquad y = -\frac{1}{2}x - 1.$$

3. Applications of linear function and linear equations

Example *The National Car Rental Company has determined that the cumulative cost of operating a vehicle is \$0.41 per mile. Write an equation that relates the cumulative cost C, in dollars, of operating a car and the number x of miles it has been driven. Find the cost of operating a car for 1000 miles.*

Solution *The cost C is a linear function of the miles x with the slope $m = 0.41$. Hence,*

$$C = 0.41x, \quad or \quad C(x) = 0.41x.$$

Then the cost of operating a car for 1000 miles is

$$C(1000) = 0.41(1000) = 410 \ dollars.$$

Example *Each Sunday a newspaper agency sells x copies of a newspaper for \$1.00 per copy. The cost to the agency of each newspaper is \$0.50. The agency pays a fixed cost for storage, delivery, and so on, of \$100 per Sunday. Write an equation that relates the profit P, in dollars, to the number x of copies sold. What is the profit to the agency if 1000 copies are sold?*

Solution *The profit P is a linear function of the number x of copies of newspaper sold with slope $1 - 0.5 = 0.5$ and y –intercept $b = -100$. Hence,*

$$P = (1.00 - 0.50)x - 100 = 0.5x - 100, \quad or \quad P(x) = 0.5x - 100.$$

The profit to the agency when 1000 copies are sold is:

$$P(1000) = 0.5(1000) - 100 = 400 \ dollars.$$