Practice Problems - Solutions
for the Calculus I/Precalculus Placement Test - Fall, 2005
Part 10

1. Evaluate the following.

$$(1)\ \sin(405^\circ) \qquad (2)\ \cos\frac{7\pi}{3} \qquad (3)\ \tan\frac{7\pi}{6}$$

(1) $\sin(405^\circ) = \sin(405^\circ - 360^\circ) = \sin 45^\circ = \dfrac{\sqrt{2}}{2}$

(2) $\cos\dfrac{7\pi}{3} = \cos\left(\dfrac{7\pi}{3} - 2\pi\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

(3) $\tan\dfrac{7\pi}{6} = \tan\dfrac{\pi}{6} = \frac{1}{3}\sqrt{3}$

2. Give the domain of $\tan x$ for $-2\pi \le x \le 2\pi$.

The domain of $\tan x$ contains all real numbers except where $\cos x = 0$.

$\cos x = 0$ when $x = \pm\dfrac{\pi}{2},\ x = \pm\dfrac{3\pi}{2},\ x = \pm\dfrac{5\pi}{2},\ \ldots$

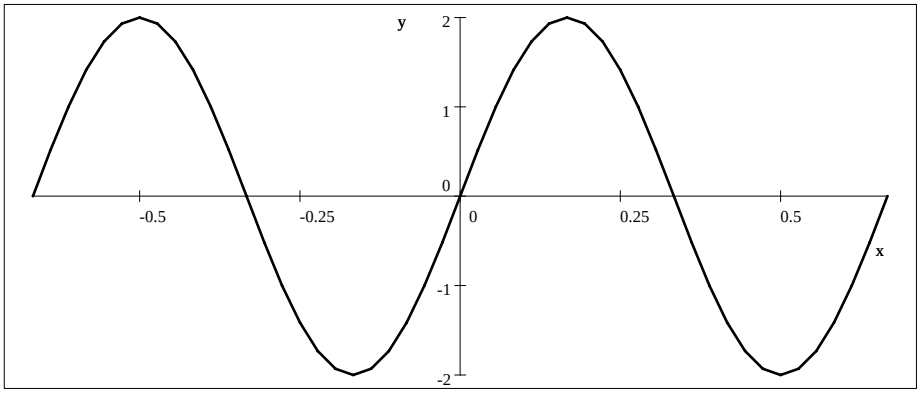
So, the domain of $\tan x$ contains all real numbers except $x = \pm\dfrac{\pi}{2},\ x = \pm\dfrac{3\pi}{2},\ x = \pm\dfrac{5\pi}{2},\ \ldots.$

3. Determine the amplitude and period of each function and then sketch its graph for two periods.

$$(1)\ f(x) = 2\sin(3\pi x) \qquad (2)\ f(x) = -3\cos\left(\frac{1}{2}x\right) \qquad (3)\ f(x) = 4\cos\left(\frac{\pi}{3}x\right)$$

(1) $f(x) = 2\sin(3\pi x)$

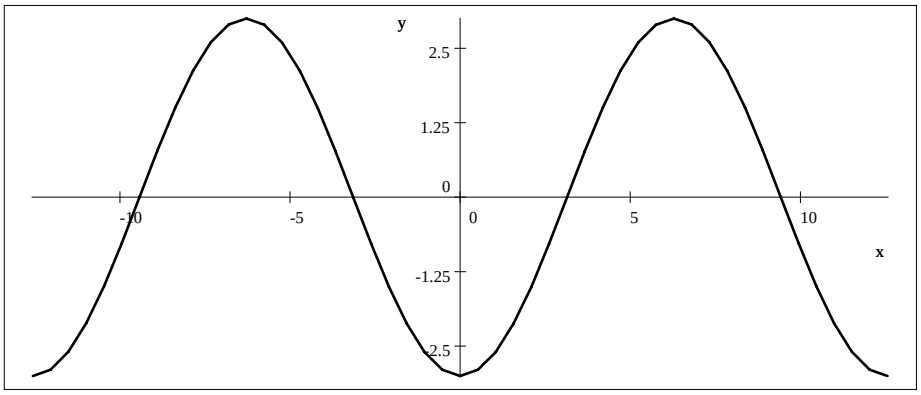
The amplitude $= 2$, the period $= \dfrac{2\pi}{3\pi} = \dfrac{2}{3}$



$y = 2\sin(3\pi x)$

(2) $f(x) = -3\cos(\frac{1}{2}x)$

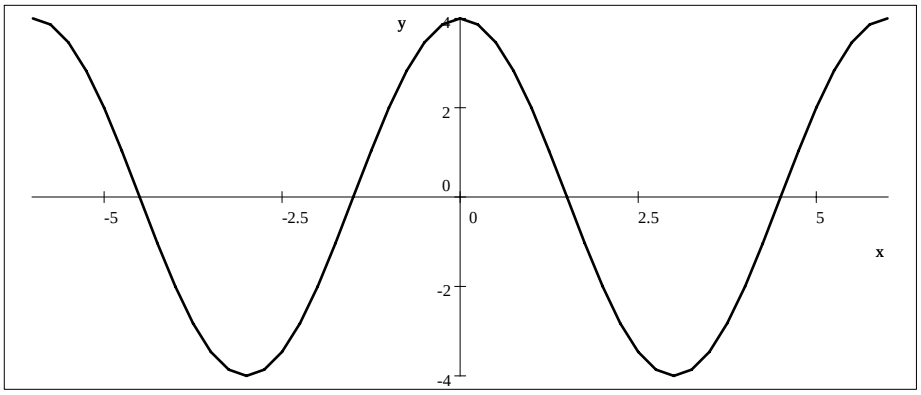
The amplitude $= 3$, the period $= \dfrac{2\pi}{\frac{1}{2}} = 4\pi$



$y = -3\cos(\frac{1}{2}x)$

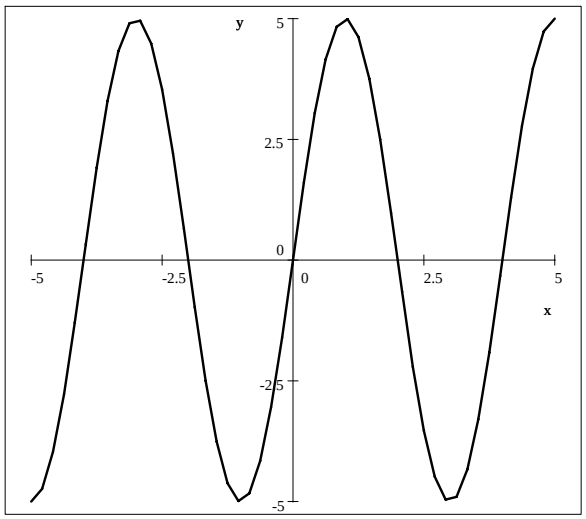
(3) $f(x) = 4\cos\left(\frac{\pi}{3}x\right)$

The amplitude $= 4$, the period $= \dfrac{2\pi}{\frac{\pi}{3}} = 6$

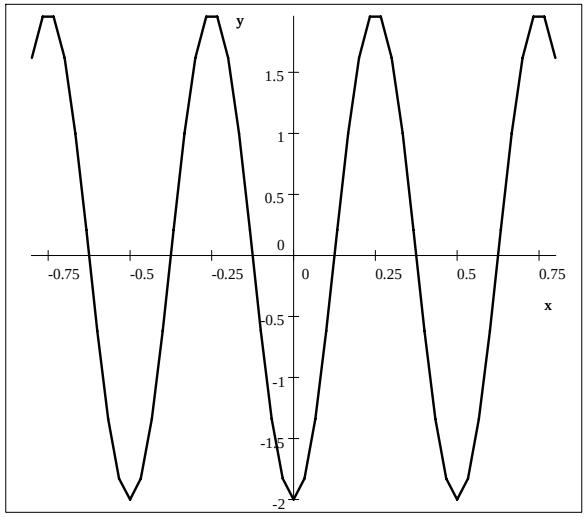


$y = 4\cos\left(\frac{\pi}{3}x\right)$

4. Graphs of $f(x) = A\sin(\omega x)$ or $f(x) = A\cos(\omega x)$ are given below. Find a function for each graph. Estimate A and ω as close as possible.



(1)



(2)

(1) It is a graph of $f(x) = A\sin(bx)$ with amplitude $= 5$, and period $= 4$.

So, $A = 5$, $\dfrac{2\pi}{\omega} = 4$, $\omega = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$, and $f(x) = 5\sin\left(\dfrac{\pi}{2}x\right)$.

(2) It is a graph of $f(x) = A\cos(bx)$ with amplitude $= 2$ and period $= 4.5$.
So, $A = -2$, $\frac{2\pi}{\omega} = 0.5$, $\omega = \frac{2\pi}{0.5} = 4\pi$, and $f(x) = -2\cos(4\pi x)$.

5. Let θ be acute $\left(0 < \theta < \frac{\pi}{2}\right)$. If $\sin\theta = \frac{2}{3}$, find $\sin(2\theta)$ and $\cos(2\theta)$.

$\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(\frac{2}{3}\right)\left(\sqrt{1-\left(\frac{2}{3}\right)^2}\right) = \frac{4}{9}\sqrt{5}$

$\cos(2\theta) = 1 - 2\sin^2\theta = 1 - 2\left(\frac{2}{3}\right)^2 = \frac{1}{9}$

6. Let θ be acute $\left(0 < \theta < \frac{\pi}{2}\right)$. If $\cos\theta = \frac{1}{4}$, find $\sin(2\theta)$ and $\cos(2\theta)$.

$\sin(2\theta) = 2\sin\theta\cos\theta = 2\sqrt{1-\left(\frac{1}{4}\right)^2}\left(\frac{1}{4}\right) = \frac{1}{8}\sqrt{15}$

$\cos(2\theta) = 2\cos^2\theta - 1 = 2\left(\frac{1}{4}\right)^2 - 1 = -\frac{7}{8}$