Practice Problems - Solutions

for the Calculus I/Precalculus Placement Test - Fall, 2005

Part 2

1. Evaluate or simplify the following expressions. Express each answer without using negative exponents.

a. $3^2 + (-4)^3 + (-2)^0 = -54$

b. $(-3x)^3 = -27x^3$

c. $-4^2 + (-2)^3 = -24$

d. $(-3)^{-2} = \frac{1}{9}$

e. $\frac{1}{3^{-2}} = 9$

f. $-2x^{-1} = -\frac{2}{x}$

g. $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

h. $\left(\frac{3}{4}\right)^{-2} = \frac{16}{9}$

i. $\frac{3^{-2}4^3}{3^3 4^{-1}} = \frac{256}{243}$

2. Simplify each expression. Express each answer without using negative exponents.

a. $\frac{3^7}{3^2} = 3^4 = 243$

b. $\frac{2x^{-3}}{x^{-5}} = 2x^{-3-(-5)} = 2x^2$

c. $\frac{x^{-2}y^3}{x^3y^{-1}} = x^{-2-3}y^{3-(-1)} = x^{-5}y^4 = \frac{y^4}{x^5}$

d. $\frac{(xy^{-1})^{-2}}{xy^3} = \frac{x^{-2}y^2}{xy^3} = x^{-2-1}y^{2-3} = x^{-3}y^{-1} = \frac{1}{x^3y}$

e. $\frac{\left(\frac{y^2}{x}\right)^2}{x^{-2}y} = \frac{y^4}{x^2}\left(\frac{1}{x^{-2}y}\right) = y^{4-1}x^{-2+2} = y^3$

3. Simplify radicals and express each answer with rational exponents.

a. $\sqrt[3]{8} = 2$

b. $\sqrt[4]{32a^5} = (32a^5)^{1/4} = (2^5)^{1/4}a^{5/4} = 2^{5/4}a^{5/4} = 2(2^{1/4})a^{5/4}$

c. $\sqrt{\frac{8x^3}{9y^6}} = \left(\frac{8x^3}{9y^6}\right)^{1/2} = \left(\frac{2^3}{3^2}\right)^{1/2}\left(\frac{x^3}{y^6}\right)^{1/2} = \frac{2}{3}(2)^{1/2}\frac{x^{3/2}}{y^3}$

d. $(xy)^{1/4}(x^2y^2)^{1/2} = x^{1/4}y^{1/4}xy = x^{1/4+1}y^{1/4+1} = x^{5/4}y^{5/4}$

e. $\left(\frac{x^{1/2}}{y^2}\right)^4\left(\frac{y^{1/3}}{x^{-2/3}}\right)^3 = \left(\frac{x^2}{y^8}\right)\left(\frac{y}{x^{-2}}\right) = x^{2+2}y^{-8+1} = \frac{x^4}{y^7}$

f. $\sqrt{3x^2}\,\sqrt{12x^3} = (3x^2)^{1/2}(12x^3)^{1/2} = 3^{1/2}(12)^{1/2}(xx^{3/2}) = (36)^{1/2}(x^{5/2}) = 6x^{5/2}$

g. $\frac{\sqrt{x^2y^4}\,\sqrt{64x^3y}}{\sqrt{81xy^6}} = \frac{(x^2y^4)^{1/2}(64x^3y)^{1/2}}{(81xy^6)^{1/2}} = \frac{xy^2(8x^{3/2}y^{1/2})}{9x^{1/2}y^3} = \frac{8}{9}x^{1+3/2-1/2}y^{2+1/2-3}$

$= \frac{8}{9}x^2y^{-1/2} = \frac{8x^2}{9y^{1/2}}$

h. $\sqrt{x} + 2\sqrt{x^3} = x^{1/2} + 2x^{3/2} = x^{1/2}(1 + 2x)$

i. $\sqrt{1 - x^2} - \dfrac{1}{\sqrt{1 - x^2}} = \dfrac{\sqrt{1 - x^2}\,\sqrt{1 - x^2} - 1}{\sqrt{1 - x^2}} = \dfrac{(1 - x^2) - 1}{\sqrt{1 - x^2}} = \dfrac{-x^2}{\sqrt{1 - x^2}}$

4. Determine whether the expression is a polynomial. If it is, give its degree.

a. $10z^5 - z$

It is a 5th degree polynomial or a polynomial of degree 5.

b. π

It is a constant or a polynomial of degree 0.

c. $2y^3 - \sqrt{2}$

It is a cubic polynomial or a polynomial of degree 3.

d. $\sqrt{x} - 2x^2$

It is not a polynomial because $\sqrt{x}$ is in it.

e. $2x^2 - \frac{1}{x}$

It is not a polynomial because $\frac{1}{x}$ is in it.

f. $x^{\pi} - 1$

It is not a polynomial because π is not an integer.

5. Write the polynomial in standard form.

a. $(x^3 + 3x^2 + 2) - (x^2 - 4x + 2) = x^3 + 2x^2 + 4x$

b. $-2(x^2 + x + 1) + 6(x^4 - x) = 6x^4 - 2x^2 - 8x - 2$

c. $x(x^2 + x - 2) = x^3 + x^2 - 2x$

d. $(x + 1)(x^2 + 2x - 4) = x^3 + 3x^2 - 2x - 4$

e. $(2x - 3)(x^2 - x + 1) = 2x^3 - 5x^2 + 5x - 3$

f. $(x - 1)^2(2x^2 - x - 1) = 2x^4 - 5x^3 + 3x^2 + x - 1$