Practice Problems - Solutions
for the Calculus I/Precalculus Placement Test - Fall, 2005

Part 8

1. Which of functions are exponential functions.

(1) $f(x) = x^4$ (2) $f(x) = (\pi)^x$ (3) $f(x) = \left(\frac{1}{e}\right)^x$ (4) $f(x) = 2^\pi$ (5) $f(x) = x^x$

$f(x) = (\pi)^x$, and $f(x) = \left(\frac{1}{e}\right)^x$ are exponential functions.

2. Let $f(x) = 3^x$ and $g(x) = \left(\frac{1}{3}\right)^x$. Evaluate and simplify if possible the following.

(1) $f(-2) + g(0)$ (2) $f(1)g(-2)$ (3) $\frac{f(-2)}{g(2)}$ (4) $(g \circ f)(0)$

(1) $f(-2) + g(0) = 3^{-2} + \left(\frac{1}{3}\right)^0 = \frac{1}{9} + 1 = \frac{10}{9}$

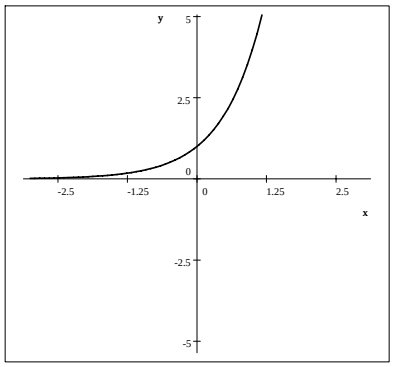
(2) $f(1)g(-2) = 3^1\left(\frac{1}{3}\right)^{-2} = 3(3^2) = 27$

(3) $\frac{f(-2)}{g(2)} = \frac{3^{-2}}{\left(\frac{1}{3}\right)^2} = \left(\frac{1}{3^2}\right)3^2 = 1$

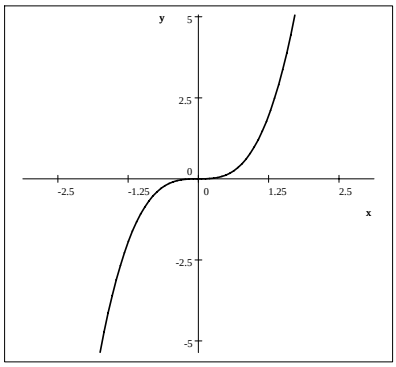
(4) $(g \circ f)(0) = g(f(0)) = g(1) = \frac{1}{3}$

3. The graphs of exponential functions are given. Match each graph to one of the function.

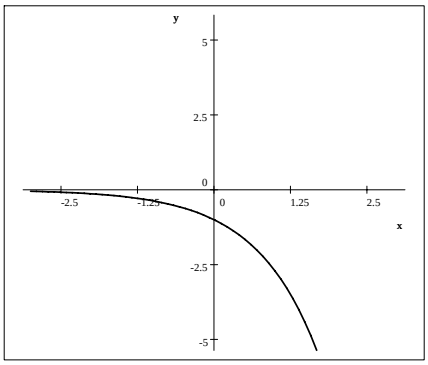
(1) $f(x) = -\left(\frac{1}{4}\right)^x$ (2) $f(x) = 4^x$ (3) $f(x) = -e^x$ (4) $f(x) = \left(\frac{1}{e}\right)^x$



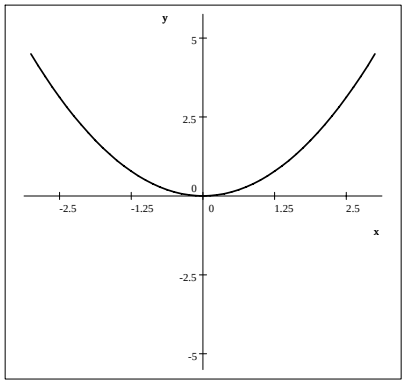
(*i*)



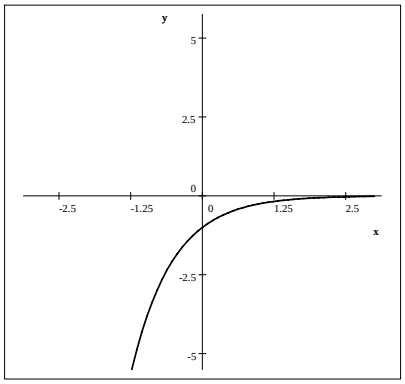
(*ii*)



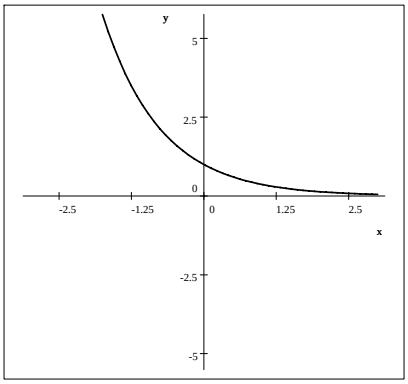
(*iii*)



(*iv*)



(*v*)



(*vi*)

(1) $f(x) = -\left(\frac{1}{4}\right)^x = -4^{-x}$, 4^{-x} is decreasing, -4^{-x} is increasing and the graph is below the x-axis $\Rightarrow (v)$

(2) $f(x) = 4^x$, 4^x is increasing and the graph is above the x –axis $\Rightarrow (i)$

(3) $f(x) = -e^x$, e^x is increasing, the graph of $-e^x$ is decreasing and below the x –axis $\Rightarrow (iii)$

(4) $f(x) = \left(\frac{1}{e}\right)^x = e^{-x}$, e^{-x} is decreasing, and the graph is above the x –axis $\Rightarrow (vi)$

4. Solve each equation.

a. $3^{x+2} = 9 = 3^2 \Leftrightarrow x + 2 = 2,\ x = 0$

b. $5^{1-2x} = \frac{1}{5} = 5^{-1},\ 1 - 2x = -1,\ -2x = -2,\ x = 1$

c. $2^{x^2-2x} = 8 = 2^3,\ x^2 - 2x = 3,\ x^2 - 2x - 3 = (x-3)(x+1) = 0,$ $\begin{matrix} x-3=0,\ x=3 \\ x+1=0,\ x=-1 \end{matrix}$

d. $(e^4)^x\left(e^{x^2}\right) = e^{12} \Leftrightarrow e^{4x+x^2} = e^{12},\ 4x + x^2 = 12,\ x^2 + 4x - 12 = (x+6)(x-2) = 0$
$x = 2,\ x = -6$

5. Convert each angle in radians to degrees.

(1) $\theta = \frac{\pi}{4}$ radians (2) $\theta = 3$ radians (3) $\theta = -\frac{5\pi}{6}$ radians (4) $\theta = \frac{11\pi}{6}$ radians

(1) $\theta = \frac{\pi}{4}$ radians $= 45^\circ$

(2) $\theta = 3$ radians $= 3\left(\frac{180}{\pi}\right) = 171.8873^\circ$

(3) $\theta = -\frac{5\pi}{6}$ radians $= -\frac{5\pi}{6}\left(\frac{180}{\pi}\right) = -150^\circ$

(4) $\theta = \frac{11\pi}{6}$ radians $= \frac{11\pi}{6}\left(\frac{180}{\pi}\right) = 330^\circ$

6. Convert each angle in degrees to radians.

(1) $\theta = 120^\circ$ (2) $\theta = -225^\circ$ (3) $\theta = 330^\circ$ (4) $\theta = 100^\circ$

(1) $\theta = 120^\circ = 120\left(\frac{\pi}{180}\right) = \frac{2}{3}\pi$

(2) $\theta = -225^\circ = -225\left(\frac{\pi}{180}\right) = -\frac{5}{4}\pi$

(3) $\theta = 330^\circ = 330\left(\frac{\pi}{180}\right) = \frac{11}{6}\pi$

(4) $\theta = 100^\circ = 100\left(\frac{\pi}{180}\right) = \frac{5}{9}\pi$

7. Find the length of arc s if the radius r of the circle and the central angle θ are given below.

a. $r = 2$ inches, $\theta = 30^\circ$

$$s = 2(30)\frac{\pi}{180} = 1.047198 \text{ inches}$$

b. $r = 3$ meters, $\theta = 120^\circ$

$$s = 3(120)\left(\frac{\pi}{180}\right) = 6.283185 \text{ meters}$$

c. $r = 1$ foot, $\theta = 360^\circ$

$$s = 2\pi(1) = 2\pi \text{ feet}$$