Mei Qin Chen

WeBWorK assignment number Homework20 is due: 11/15/2011 at 09:30am EST.

The
(* replace with url for the course home page *)
for the course contains the syllabus, grading policy and other information.

This file is /conf/snippets/setHeader.pg you can use it as a model for creating files which introduce each problem set.

The primary purpose of WeBWorK is to let you know that you are getting the correct answer or to alert you if you are making some kind of mistake. Usually you can attempt a problem as many times as you want before the due date. However, if you are having trouble figuring out your error, you should consult the book, or ask a fellow student, one of the TA's or your professor for help. Don’t spend a lot of time guessing – it’s not very efficient or effective.

Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2^3$ instead of 8, $\sin(3 \cdot \pi/2)$ instead of -1, $e^{ln(2)}$ instead of 2, $(2 + \tan(3)) \cdot (4 - \sin(5)) \cdot 6 - 7/8$ instead of 27620.3413, etc. Here’s the list of the functions which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

1. (1 pt) Library/Michigan/Chap16Sec5/Q25.pg
Write a triple integral including limits of integration that gives the volume of the cap of the solid sphere $x^2 + y^2 + z^2 \leq 32$ cut off by the plane $z = 4$ and restricted to the first octant. (In your integral, use theta, rho, and phi for $\theta$, $\rho$ and $\phi$, as needed.)

What coordinates are you using? ______
(Enter cartesian, cylindrical, or spherical.)

With $a = _____$, $b = _____$,
$c = _____$, $d = _____$,
$e = _____$, $f = _____$,
Volume $= \int_a^b \int_c^d \int_e^f _____ \ d _____ \ d _____ \ d _____$

SOLUTION
We can set this up in any of the three coordinate systems, but it is easiest in cylindrical. In cylindrical, we have

$V = \int_0^{\pi/2} \int_0^4 \int_0^{\sqrt{32-r^2}} r \ dz \ dr \ d\theta$.

In cartesian, we get

$V = \int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{\sqrt{32-x^2-y^2}} 1 \ dz \ dy \ dx$,

and in spherical,

$V = \int_0^{\pi/2} \int_0^{\arccos(4/\sqrt{32})} \int_{4/\cos\phi}^{\sqrt{32}} \rho^2 \sin\phi \ d\rho \ d\phi \ d\theta$.

Correct Answers:
- cylindrical
- $0$
- $\pi/2$
- $4$
- $4$
- $\sqrt{32-r^2}$
- $r$
- $z$
- $r$
- $\theta$

2. (1 pt) Library/FortLewis/Calc3/16-5-Spherical-integrals/HGM4-16-5-04-Spherical-integrals.pg
Suppose $f(x,y,z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ and $W$ is the bottom half of a sphere of radius 4. Enter $\rho$ as rho, $\phi$ as phi, and $\theta$ as theta.

(a) As an iterated integral,

$\int_W f \ dV = \int_A^B \int_C^D \int_E^F _____ \ d\rho \ d\phi \ d\theta$

with limits of integration

A = _____
B = _____
C = _____
D = _____
E = _____
F = _____

(b) Evaluate the integral. ________________

Correct Answers:
- $\rho \sin(\phi)$
- 0
- $2\pi$
- $\pi/2$
- $\pi$
- 0
- 4
- $\pi \cdot 4^2$

Suppose the solid $W$ in the figure consists of the points below the xy-plane that are between concentric spheres centered at the origin of radii 4 and 6. Find the limits of integration for an iterated integral of the form

$$\iiint_W f\,dV = \int_A^B \int_C^D \int_E^F f(\rho, \phi, \theta) \rho^2 \sin(\phi)\,d\rho\,d\phi\,d\theta.$$ 

A = ________
B = ________
C = ________
D = ________
E = ________
F = ________

If necessary, enter $\rho$ as rho, $\phi$ as phi, and $\theta$ as theta.

(Drag to rotate)

Correct Answers:
- 0
- 6.28319
- 1.5708
- 3.14159
- 4
- 6

4. (1 pt) Library/ASU-topics/setCalculus/stef/stef15_8p1.pg

Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 36$, above the xy plane, and outside the cone $z = 7 \sqrt{x^2 + y^2}$.

Correct Answers:
- 447.842600166363

5. (2 pts) Library/272/setStewart16_1/UR_VC_11_2.pg

Compute the gradient vector fields of the following functions:

A. $f(x,y) = 1x^2 + 8y^2$

$\nabla f(x,y) = _____i + _____j$

B. $f(x,y) = x^2 + 2y^2$

$\nabla f(x,y) = _____i + _____j$

C. $f(x,y) = 1x + 8y$

$\nabla f(x,y) = _____i + _____j$

D. $f(x,y,z) = 1x + 8y + 9z$

$\nabla f(x,y) = _____i + _____j + _____k$

E. $f(x,y,z) = 1x^2 + 8y^2 + 9z^2$

$\nabla f(x,y) = _____i + _____j + _____k$

Correct Answers:
- $2*1*x$
- $2*8*y$
- $(9*x^2)(9 - 1) * y^\top(2)$
- $(2*y^2)(2 - 1) * x^\top(2)$
- 1
- 8
- 1
- 8
- 9
- $2*1*x$
- $2*8*y$
- $2*9*z$

6. (1 pt) Library/Michigan/Chap17Sec3/Q27.pg

Match the vector fields with the graphs below.

(a) $\vec{F}_1 = yi + xj$ matches graph __?

(b) $\vec{F}_2 = -yi + xj$ matches graph __?

(c) $\vec{F}_3 = xi - yj$ matches graph __?

(d) $\vec{F}_4 = yi - xj$ matches graph __?

For all graphs, vectors are shown as line segments, with a dot at the tail of the vector.

SOLUTION

To see which graph is which, consider points on the x- and y-axes. On the x-axis, $y = 0$, and on the y-axis, $x = 0$

(a) $\vec{F}_1(0,y) = yi$, a vector pointing right if $y > 0$ and left if $y < 0$, while $\vec{F}_1(x,0) = xj$, a vector pointing up if $x > 0$ and down if $x < 0$. Thus $\vec{F}_1(x,y)$ is given by 4.

(b) $\vec{F}_2(0,y) = -yi$, a vector pointing left if $y > 0$ and down if $y < 0$, while $\vec{F}_2(x,0) = xj$, a vector pointing up if $x > 0$ and down if $x < 0$. Thus $\vec{F}_2(x,y)$ is given by 1.

(c) $\vec{F}_3(0,y) = -yj$, a vector pointing down if $y > 0$ and up if $y < 0$, while $\vec{F}_3(x,0) = xj$, a vector pointing right if $x > 0$ and left if $x < 0$. Thus $\vec{F}_3(x,y)$ is given by 2.

(d) $\vec{F}_4(0,y) = yi$, a vector pointing right if $y > 0$ and left if $y < 0$, while $\vec{F}_4(x,0) = -xj$, a vector pointing down if $x > 0$ and up if $x < 0$. Thus $\vec{F}_4(x,y)$ is given by 3.

Correct Answers:
- 4
- 1
7. (3 pts) Library/ASU-topics/setCalculus/stef16_3p1.pg
For each of the following vector fields \( \mathbf{F} \), decide whether it is conservative or not by computing the appropriate first order partial derivatives. Type in a potential function \( f \) (that is, \( \nabla f = \mathbf{F} \)) with \( f(0,0) = 0 \). If it is not conservative, type N.

A. \( \mathbf{F}(x,y) = (14x - 6y)i + (-6x + 10y)j \)
\( f(x,y) = \) 

B. \( \mathbf{F}(x,y) = 7yi + 8xj \)
\( f(x,y) = \)

C. \( \mathbf{F}(x,y) = (7 \sin y)i + (-12y + 7x \cos y)j \)
\( f(x,y) = \)

Note: Your answers should be either expressions of \( x \) and \( y \) (e.g. “3xy + 2y”), or the letter “N”

Correct Answers:

- 7*x^2-6*x*y+5*y^2
- N
- 7*x*sin(y)-6*y^2